Superconductivity: Lecture 1
Introduction

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(Since December 2022)

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The Netherlands

Leiden Institute of Physics

Some practical matters:

Grading: Exam (70%) + Homework & presentation (30%) + 5% bonus for excellent performance

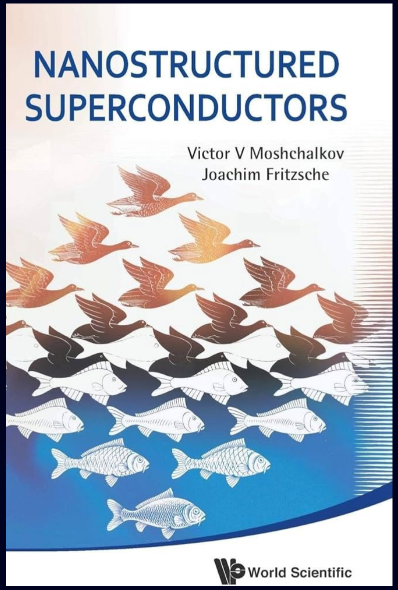
Presentations: On a recent paper (there will be a list on BS), Usually done in pairs.

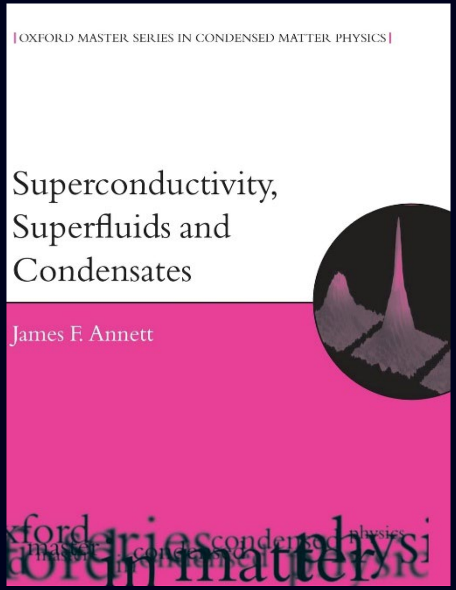
This course follows no specific text book!! So, how to find study material?

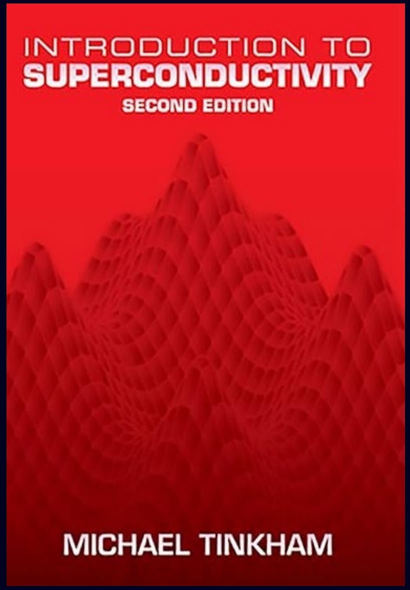
Google it! Ask chat GPT, consult your local shaman, etc. Use whatever recourses you can find!

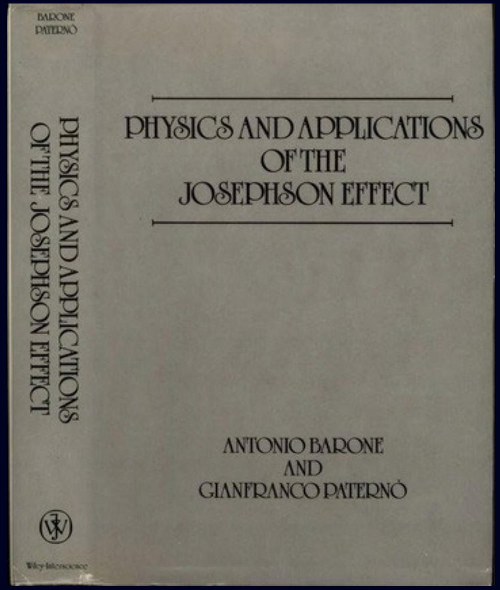
Make sure to **read the papers** mentioned in the slides, homework and presentations.

Some recommendations:



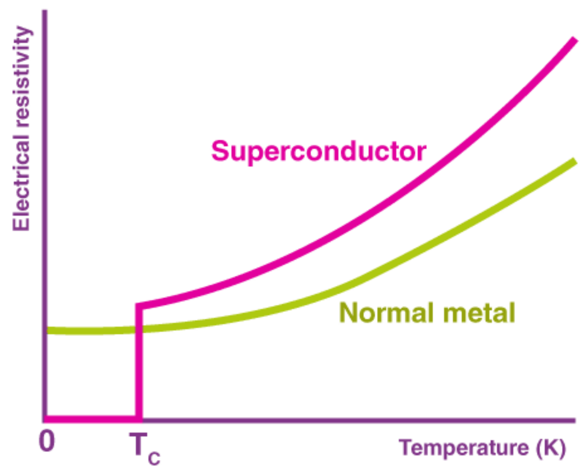






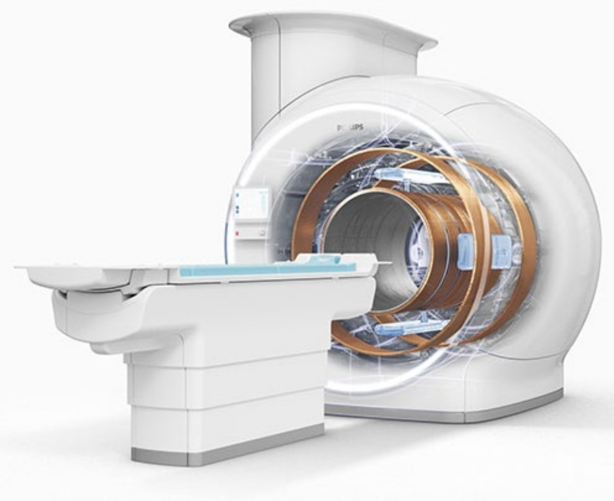
Superconductivity: How it is usually introduced

Zero resistance!



nice cables!

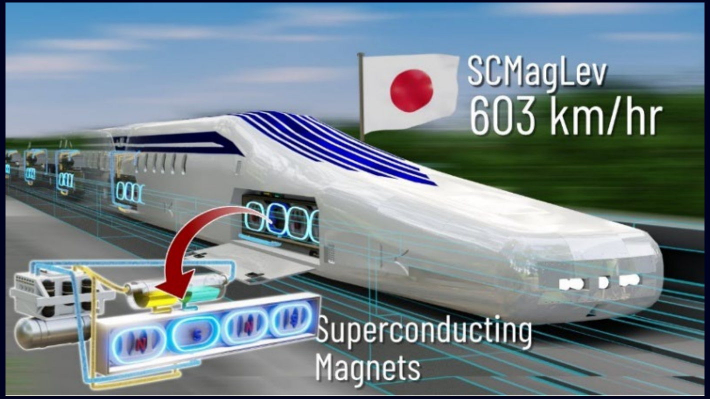
Useful for MRI & stuff



Expulsion of magnetic field

OMG they float!

Fancy levitating Japanese trains!



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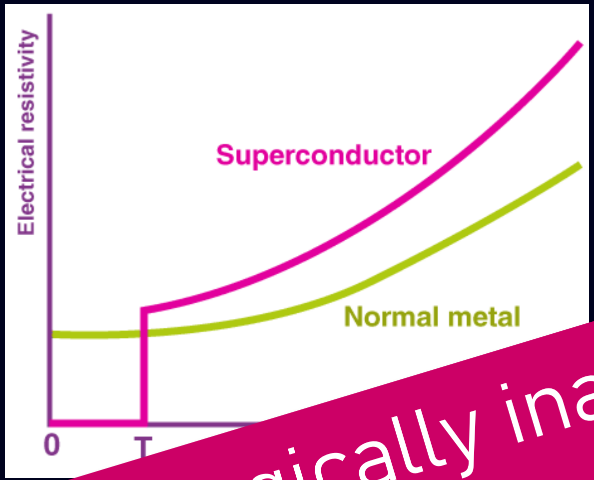

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Tragically inaccurate representation of the phenomenon (& the research field)

netic field

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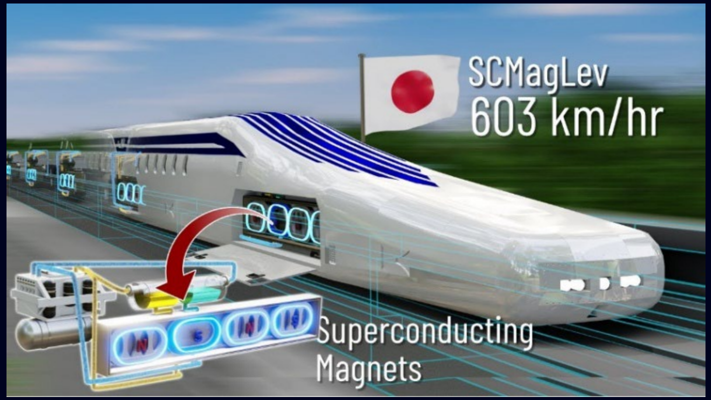
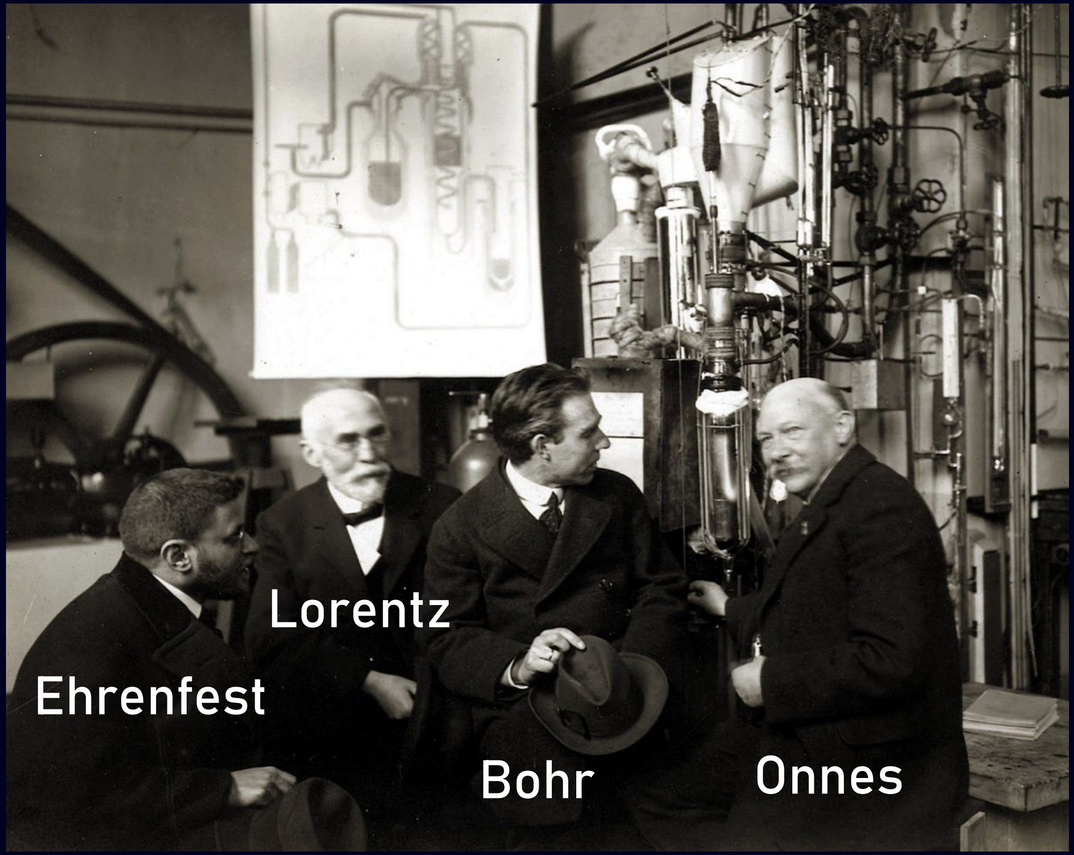
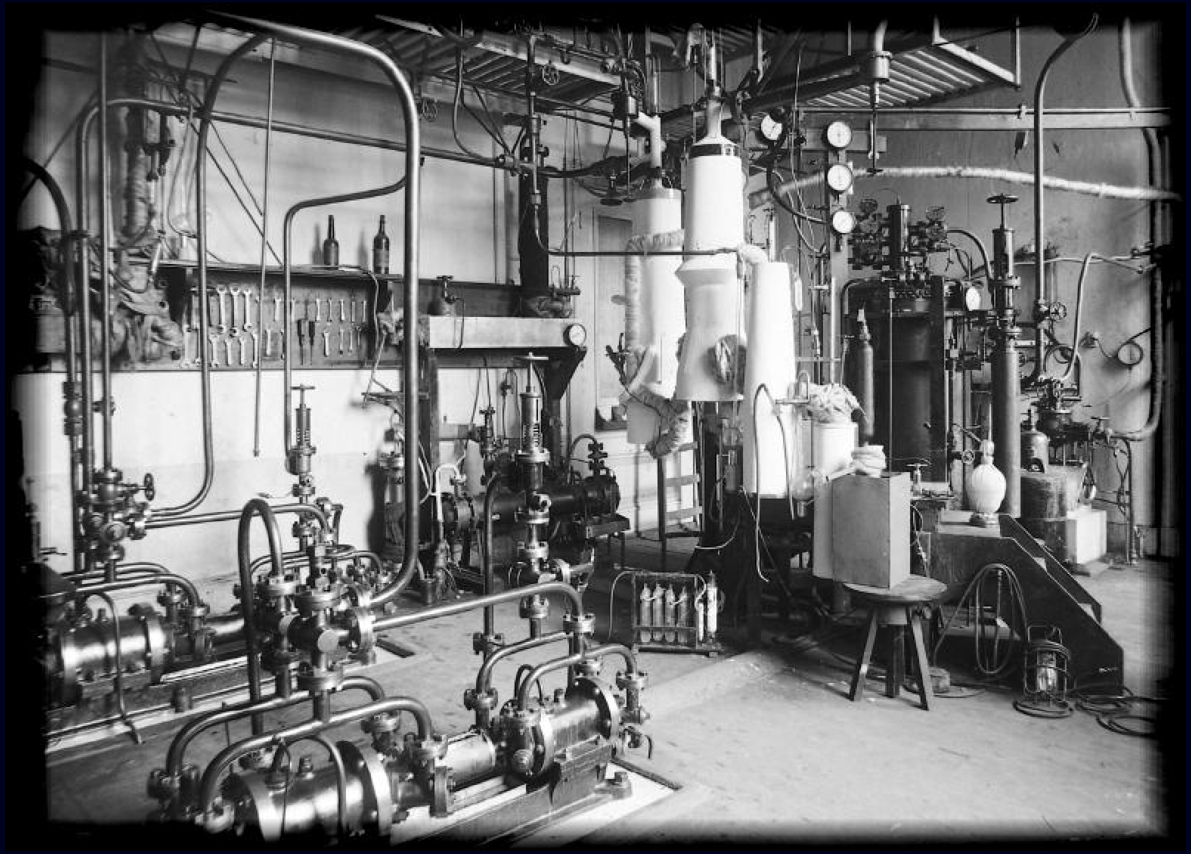


Leiden: Where it all began (and continues to this day!)





Kamerlingh Onnes Laboratory (1900s)

Timeline

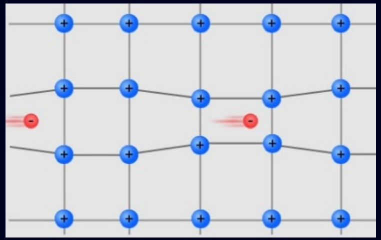
1911 Kamerlingh Onnes (Leiden) 

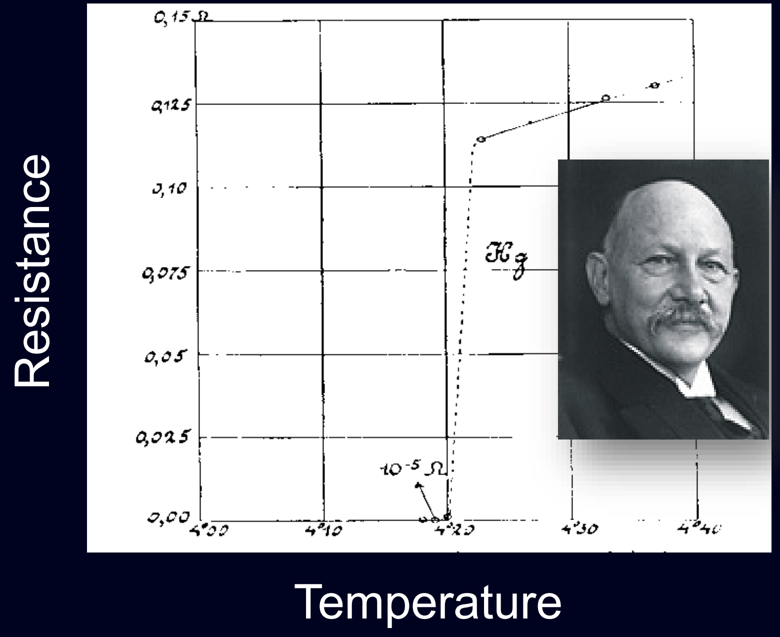
1933 Meissner effect

1950 Ginzburg - Landau theory

1957 Abrikosov vortex, BCS theory 

1962 Josephson effect 

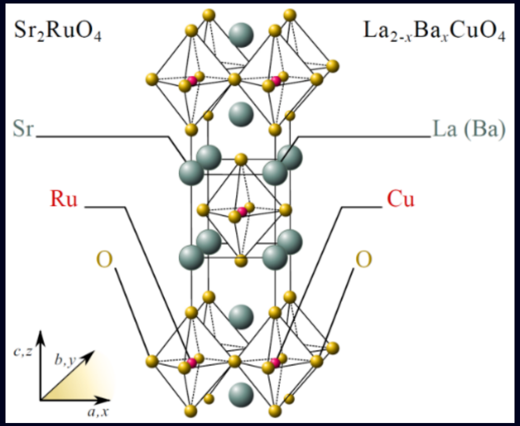


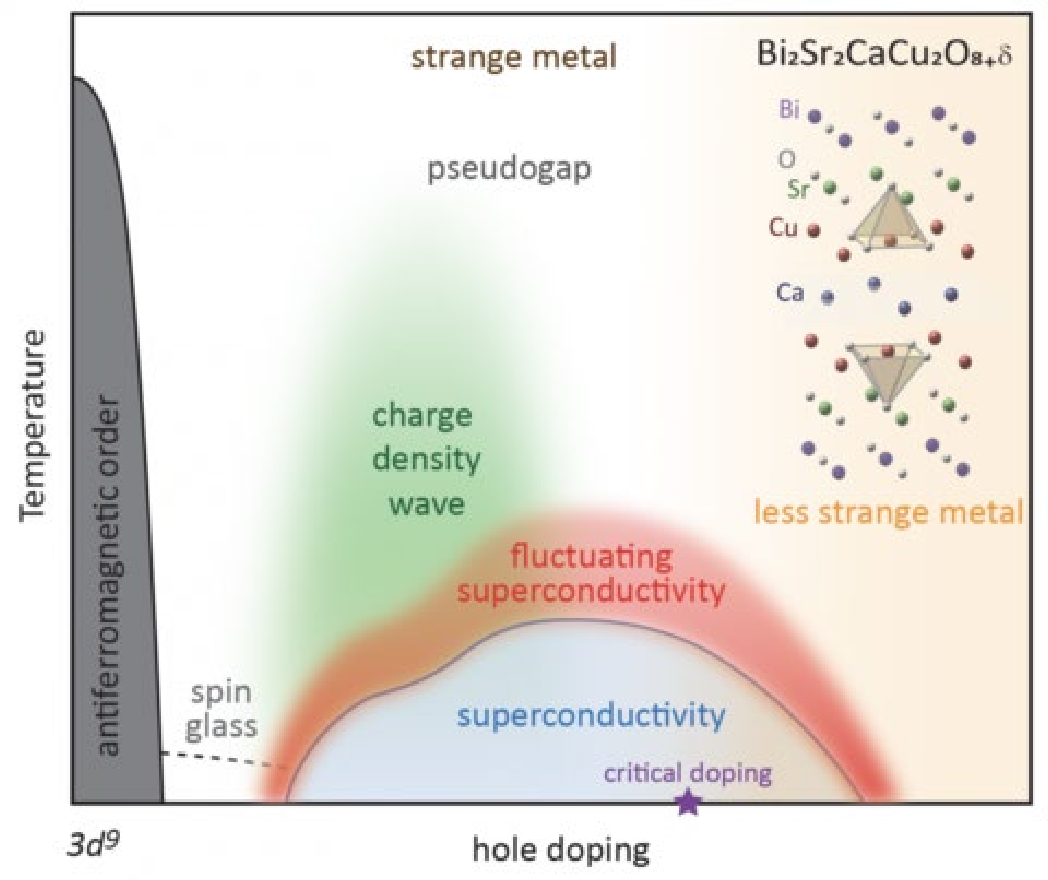


1986-now Unconventional superconductors

High-temperature SCs	$YBa_2Cu_3O_7$ 
Heavy Fermions	$CeCu_2Si_2$
Iron-based	LaFeAsO
Organic molecules:	$(BEDT\text{-}TTF)_2X$
Twisted bilayer graphene	(2018)

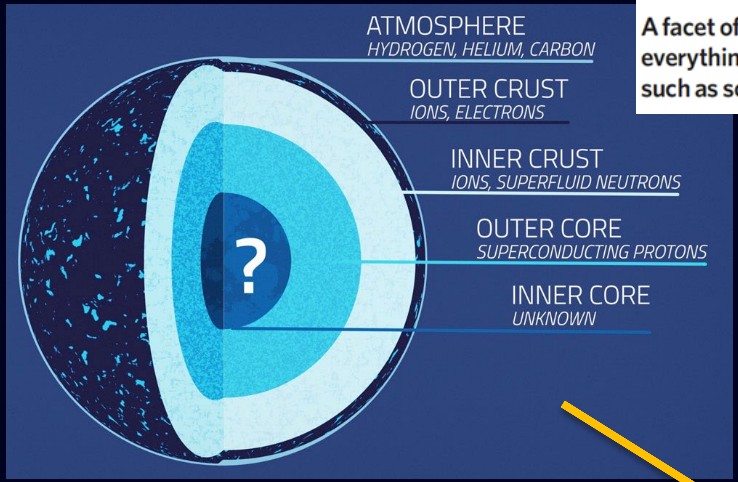
More on the way! ;-)





strange metal
Bi2Sr2CaCu2O8+δ
Bi
O
Sr
Cu
Ca
pseudogap
charge
density
wave
less strange metal
fluctuating
superconductivity
superconductivity
spin
glass
antiferromagnetic order
Temperature
critical doping
3d9
hole doping

Neutron stars are superconductors



THEORETICAL PHYSICS

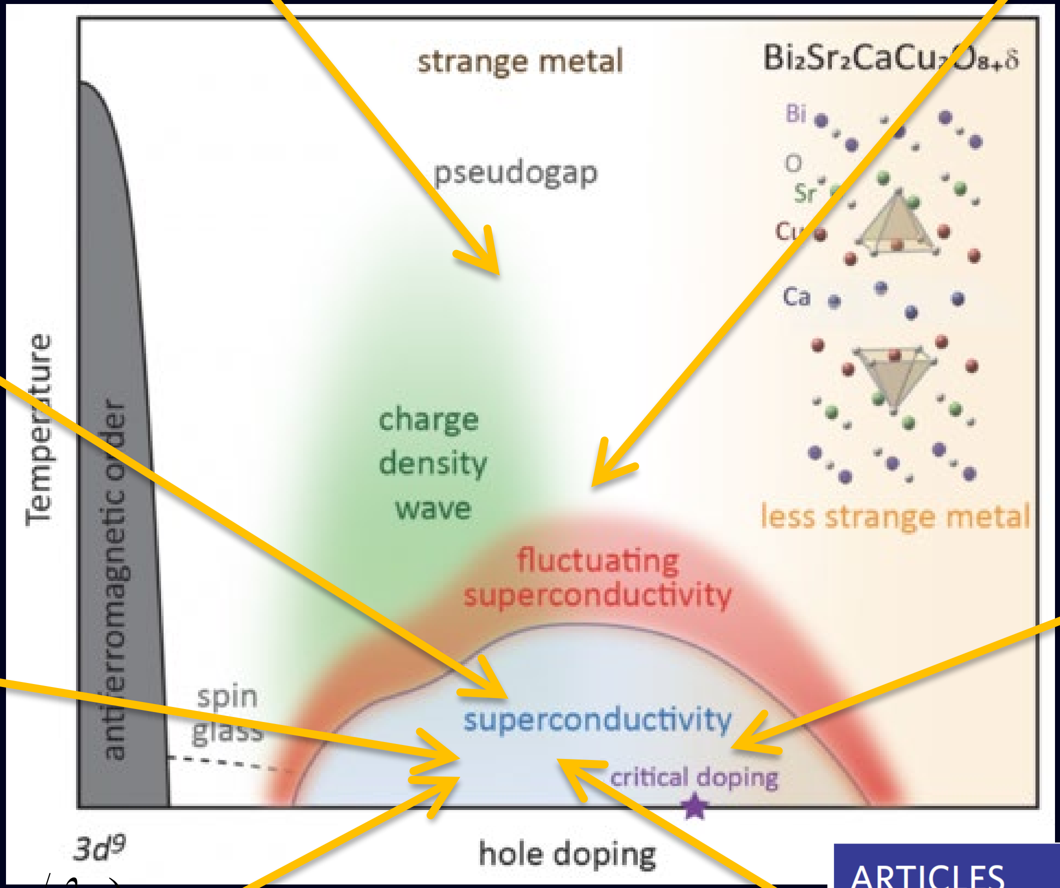
A black hole full of answers

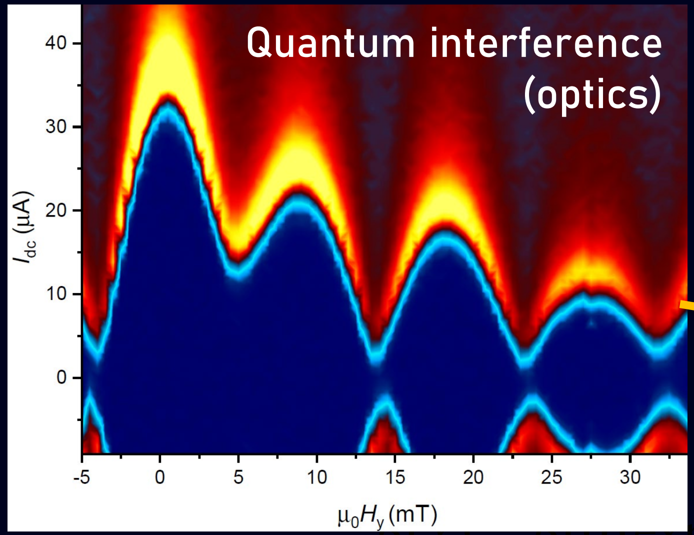
Jan Zaanen

A facet of string theory, the currently favoured route to a 'theory of everything', might help to explain some properties of exotic matter phases such as some peculiarities of high-temperature superconductors.

Article

Traversable wormhole dynamics on a quantum processor



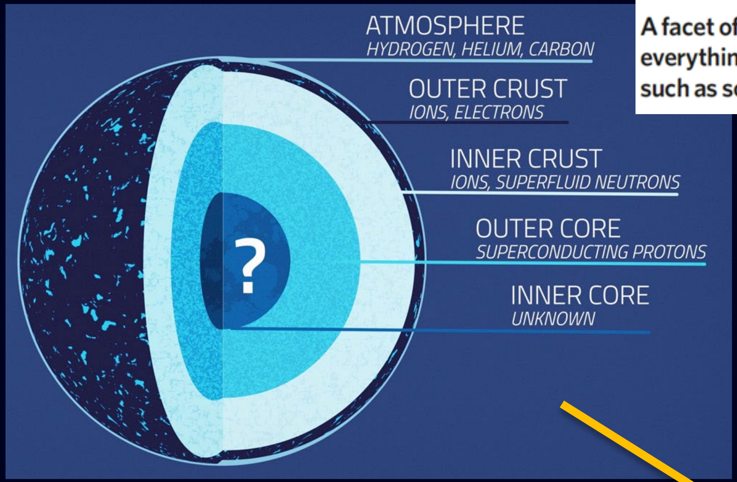


Signatures of Majorana Fermions in Hybrid Superconductor-Semiconductor Nanowire Devices

ARTICLES
PUBLISHED ONLINE: 26 JANUARY 2015 | DOI: 10.1038/NPHYS3227
nature physics

The Higgs mode in disordered superconductors close to a quantum phase transition

Neutron stars are superconductors



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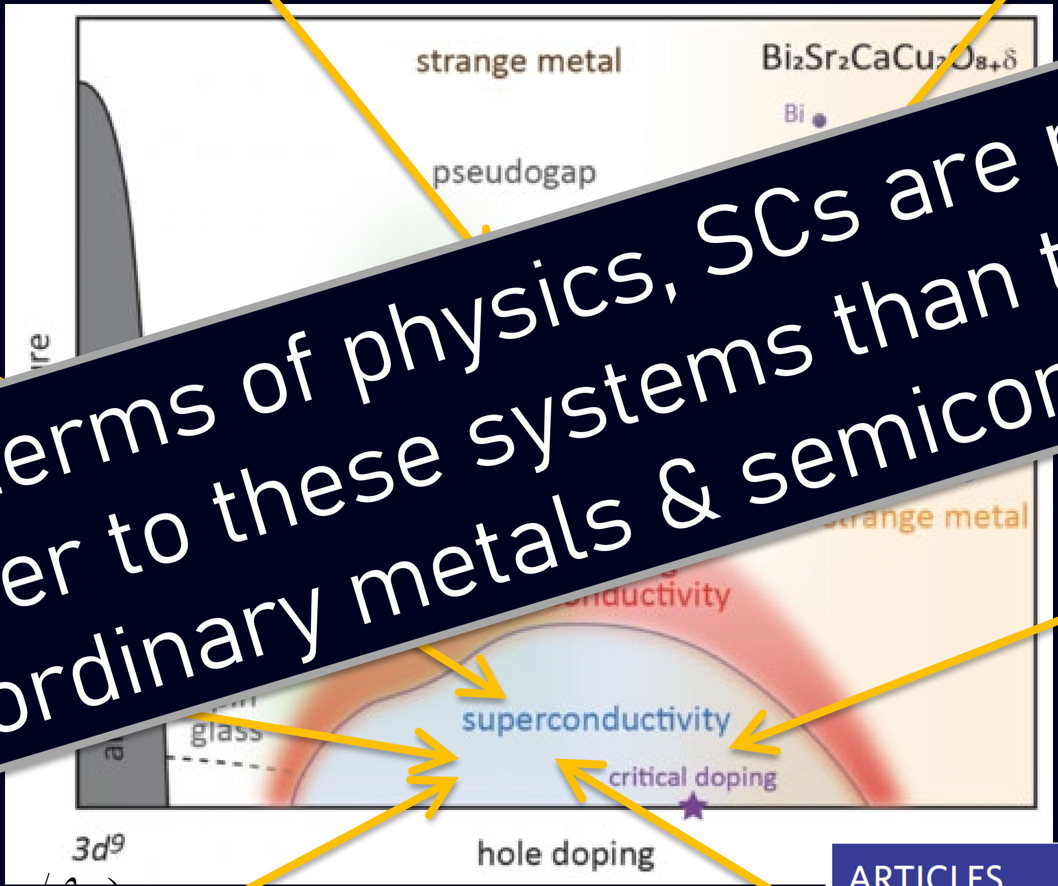

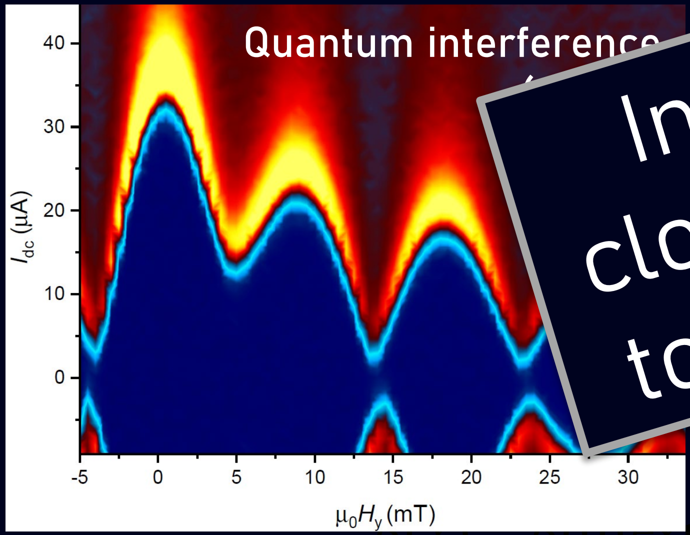

In terms of physics, SCs are much closer to these systems than they are to ordinary metals & semiconductors

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Size effect: Recall the particle in a box from QM

Quantization and confinement in normal matter

$$\frac{1}{2m}\left(-\imath\hbar\boldsymbol{\nabla} - e\boldsymbol{A}\right)^2\psi + U\psi = E\psi$$

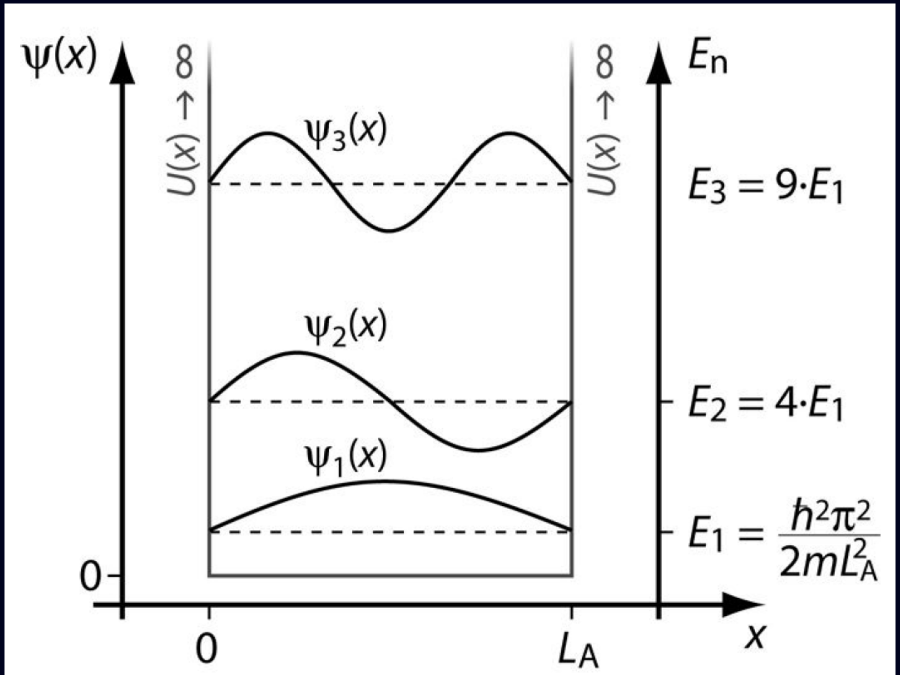
$$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2(2\pi/\lambda_n)^2}{2m} = \frac{\hbar^2\pi^2}{2mL_A^2}n^2$$

Table 1.1 Confinement by the infinite potential well

Confinement length L_A	Energy E_1	Temperature T [a]
1 Å	38 eV	
1 nm	0.38 eV	
1 μm	0.38 μeV	

[a] The corresponding temperature at which the kinetic energy of a classical 1D-particle is equal to E_1, given by Eq. (1.1).

Particle in an infinite potential well of size L_A



$$n\lambda_n/2 = L_A$$

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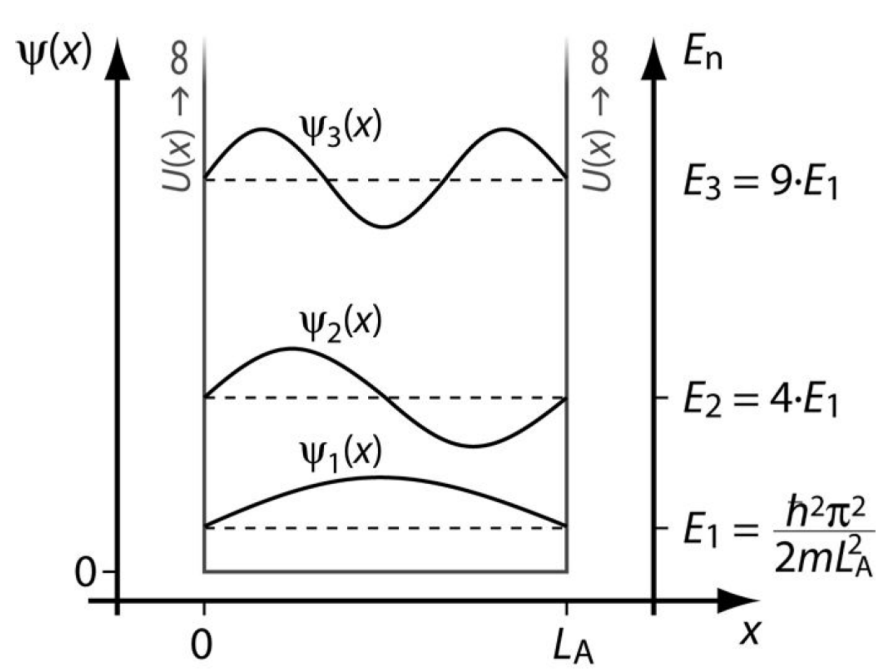


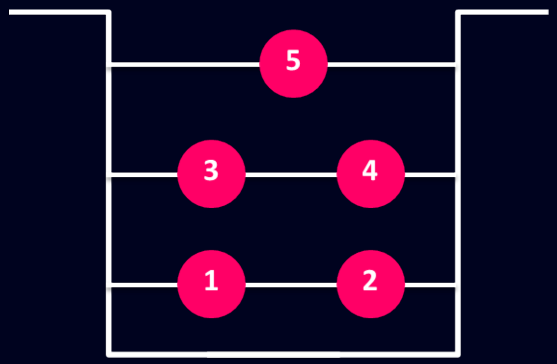
$$n\lambda_n/2=L_A$$

Normal materials:

Physical properties dominated by fermi electrons (spin ½ = Fermions).

Fermions must have unique quantum numbers (they are **individual particles**).

Wavefunction is determined by the number of particles.



$$\Psi \text{ (fermi electrons in a cm}^3\text{ metal)} \sim \Psi\,(e_1, e_2, e_3, \ldots, e_{10^{23}})$$

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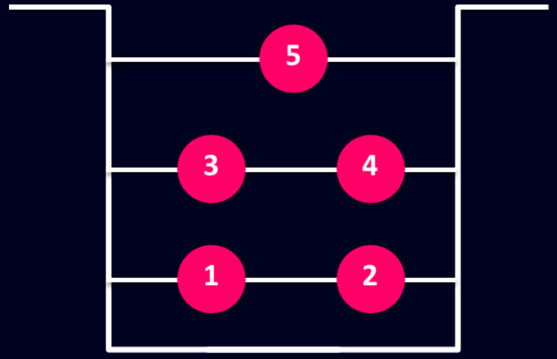
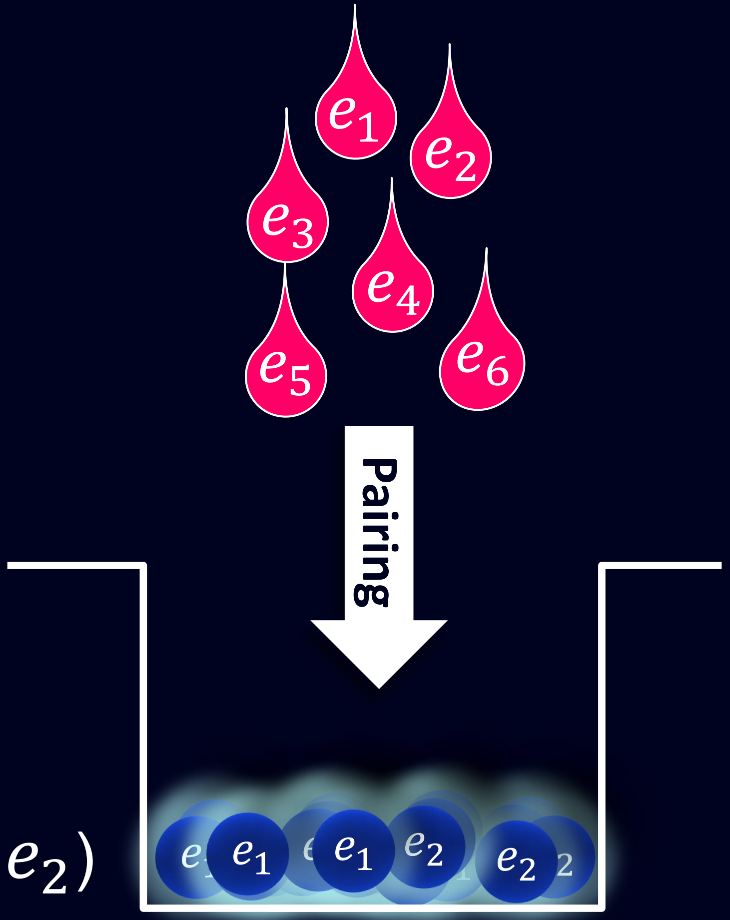


$$\Psi \text{ (fermi electrons in a cm}^3\text{ metal)} \sim \Psi\,(e_1, e_2, e_3, \ldots, e_{10^{23}})$$

Superconductors: Bosonic condensates

All particles live in a *single macroscopic state*

$$\Psi_{condensate} \text{ (}\sim 10^{23}\text{ paired electrons / cm}^3\text{)} = \Psi\,(e_1, e_2)$$



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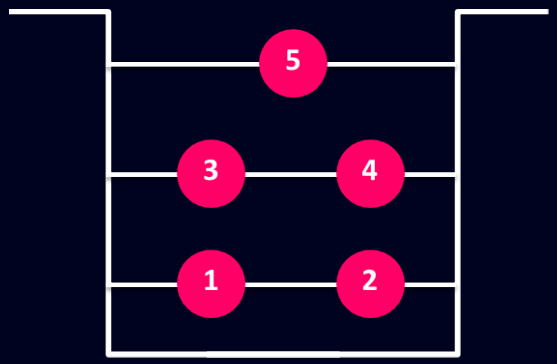
Ψ (fermi electrons in a cm^3 metal) $\sim \Psi\,(e_1, e_2, e_3, \ldots, e_{10^{23}})$



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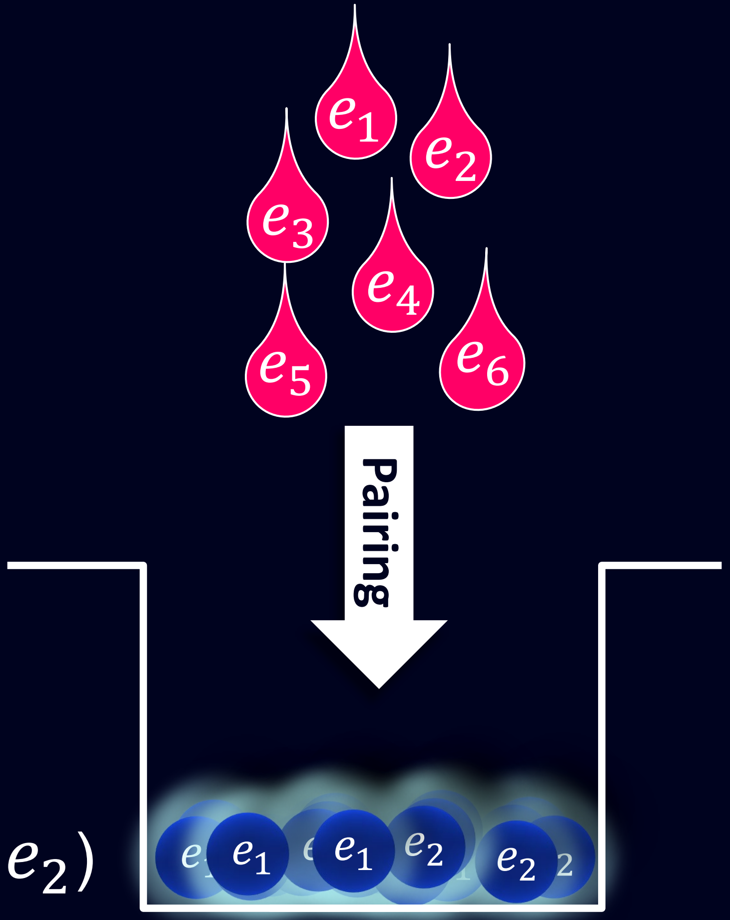

$\Psi\,(e_1, e_2)$

Don't think of it as a collection of individual pairs/particles:

All paired electrons become a single macroscopic entity

Normal materials:

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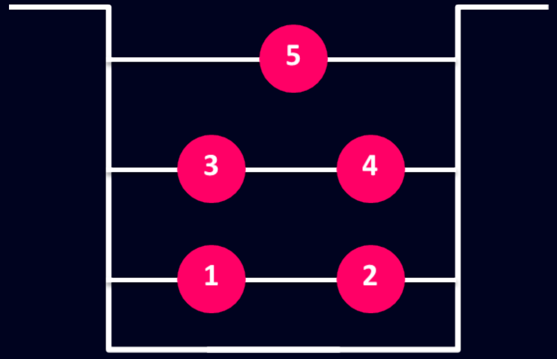


Superconductors: Bosonic condensates

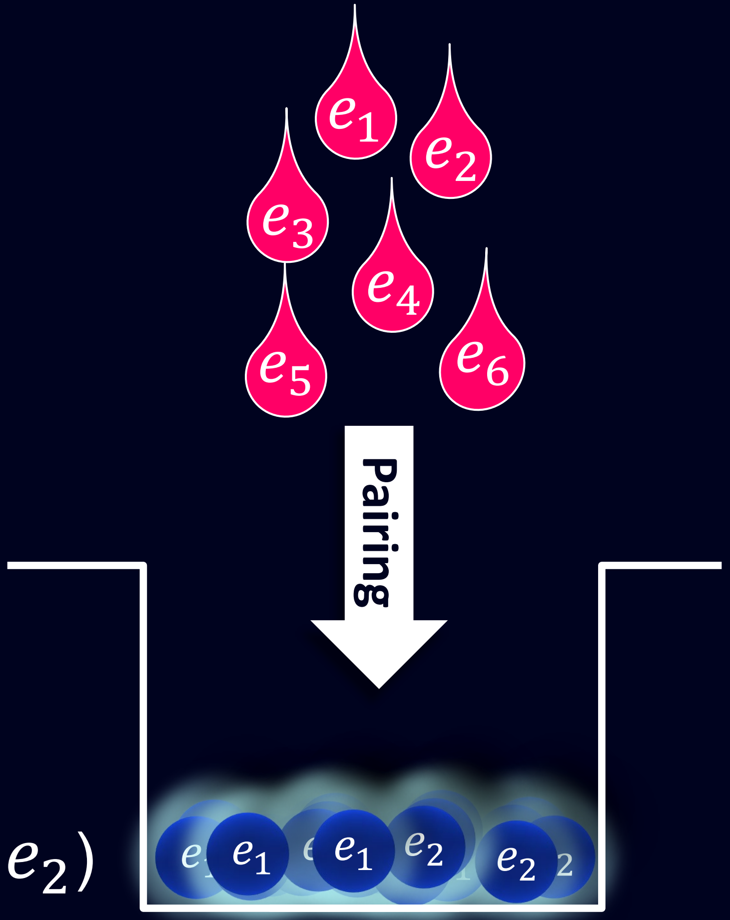
All particles live in a *single macro*...

$\Psi_{condensa}$ ... (e_1, e_2)

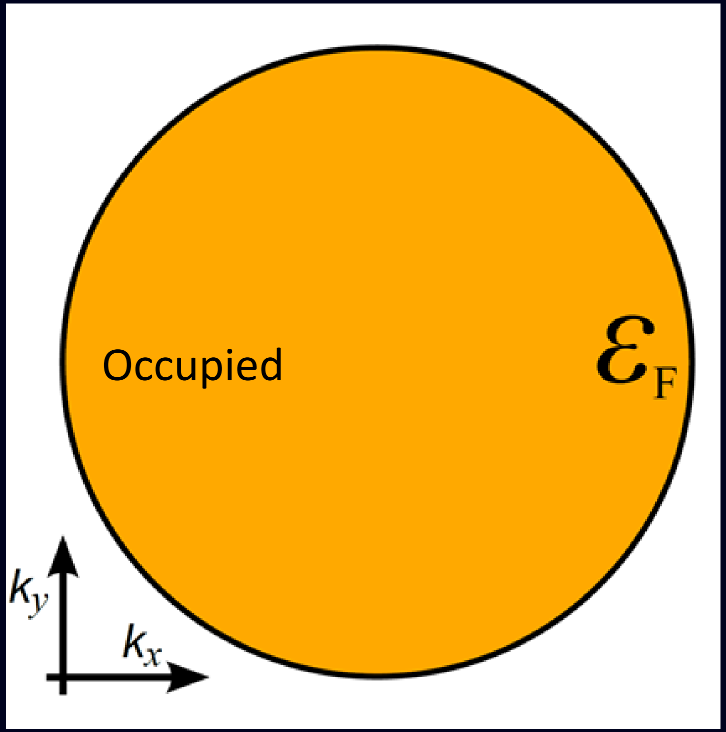
Do all electrons in the material turn into Cooper pairs??

Don't think ... as a collection of individual pairs/particles:

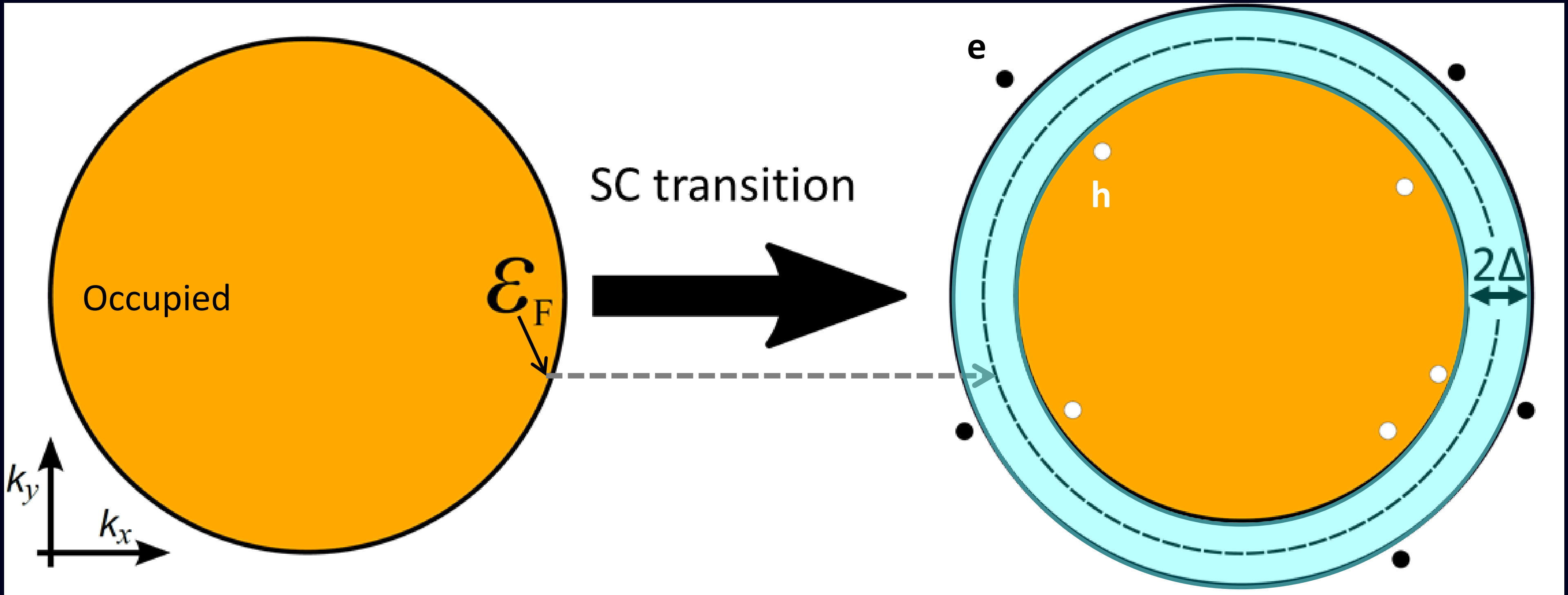
All paired electrons become a single macroscopic entity



$\Psi\,(e_1, e_2)$



Normal (T>Tc)

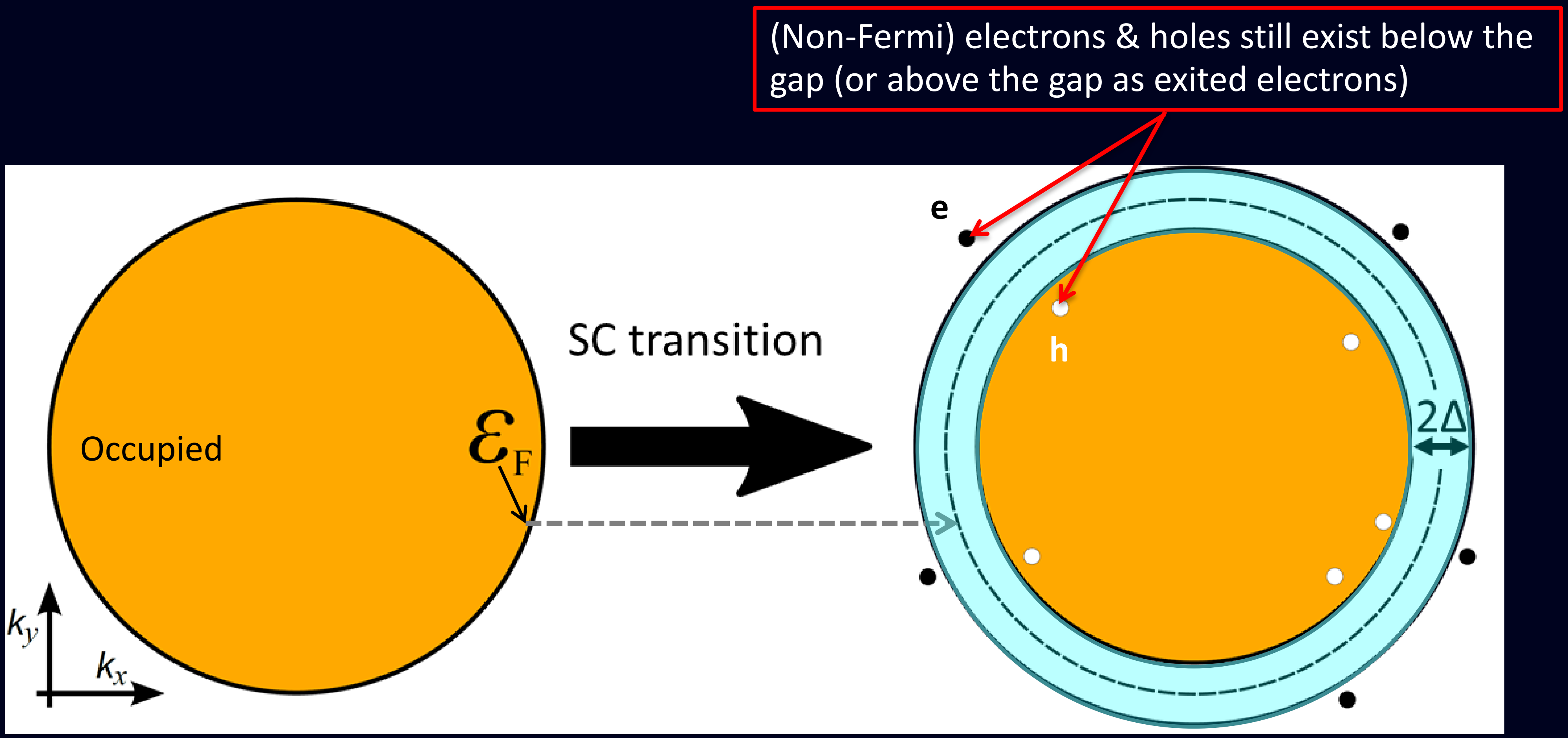


Normal (T>Tc)

SC (T<Tc)

Going forward: Forget your fermionic physics

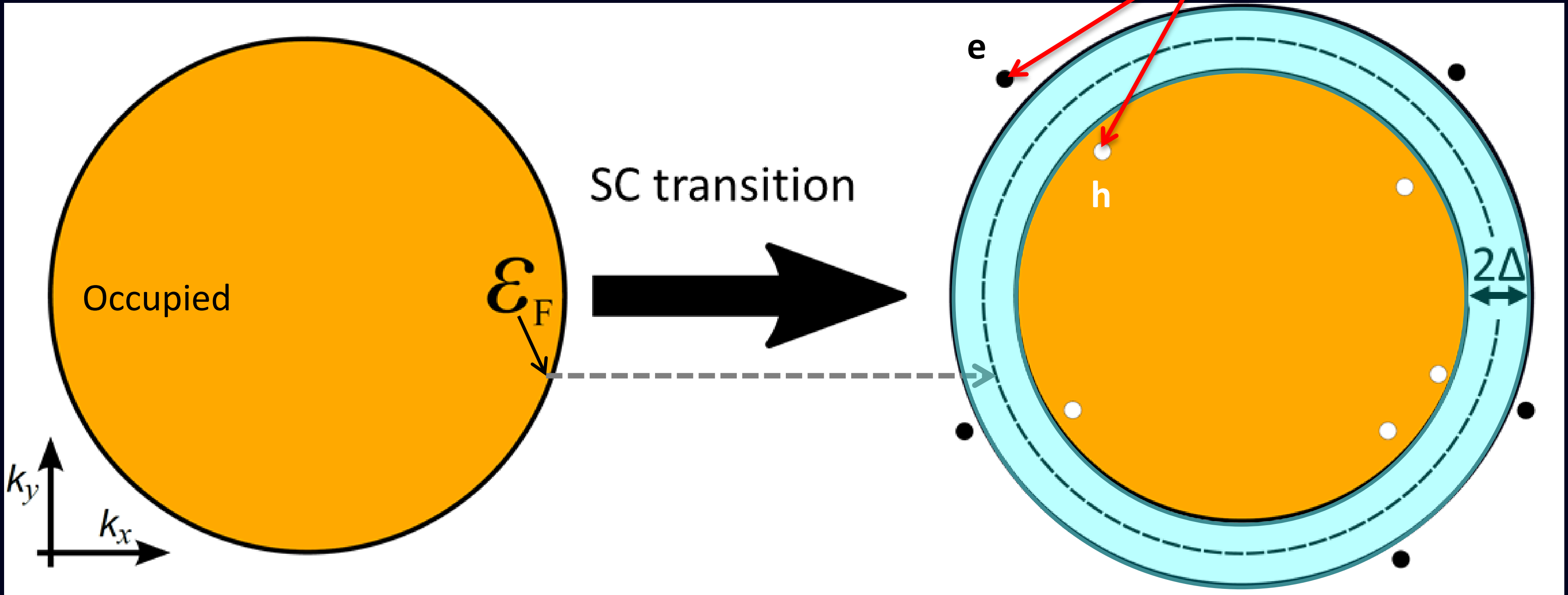
Think like a bosonic condensate!



Normal (T>Tc)

SC (T<Tc)

(Non-Fermi) electrons & holes still exist below the gap (or above the gap as exited electrons)



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Elegance of superconductors: Macroscopic wavefunction

Schrödinger

$$\frac{1}{2m}\left(-\imath\hbar\boldsymbol{\nabla} - e\boldsymbol{A}\right)^2\psi + U\psi = E\psi$$

$$\Psi = |\Psi(\mathrm{r})|e^{i\varphi(r)}$$

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Ginzburg-Landau equations

$$\frac{1}{2m^\star}(-\imath\hbar\boldsymbol{\nabla} - e^\star\boldsymbol{A})^2\psi_s + \beta|\psi_s|^2\psi_s = -\alpha\psi_s$$

$\boldsymbol{\nabla} \times \boldsymbol{A}$ (vector potential) $= \mu_0\boldsymbol{h}$ (magnetic field)

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$$e^* = 2e$$

$$m^* = 2m_e$$

$$-\alpha \leftrightarrow E$$

$$-\alpha = \frac{\hbar^2}{2m^\star\ \xi^2(T)}$$

$$\xi(T) = \frac{\xi(0)}{\sqrt{1-\frac{T}{T_{c0}}}}$$

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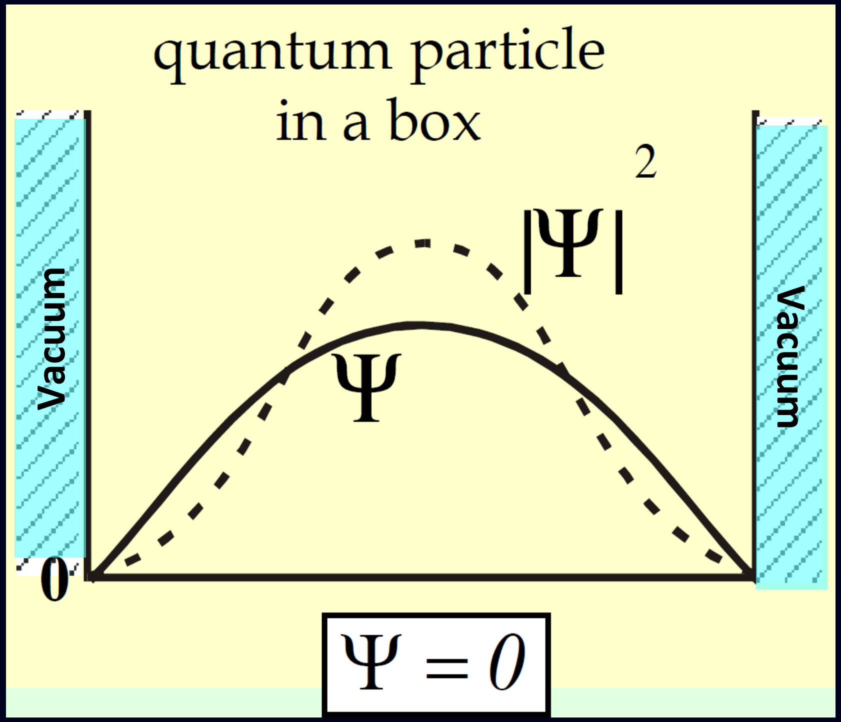
$$\xi(T) = \frac{\xi(0)}{\sqrt{1 - \frac{T}{T_{c0}}}}$$

$$\boldsymbol{j} = \boldsymbol{\nabla} \times \boldsymbol{h} = \frac{e^\star}{2m^\star}\left[\psi_s^\star(-\imath\hbar\boldsymbol{\nabla} - e^\star\boldsymbol{A})\psi_s + \psi_s(\imath\hbar\boldsymbol{\nabla} - e^\star\boldsymbol{A})\psi_s^\star\right]$$

Boundary conditions: particle in a box vs confined SC

N/vacuum
(Dirichlet boundary condition)



Density is zero at the boundary

MORE DETAIL IN MOSHCHALKOV

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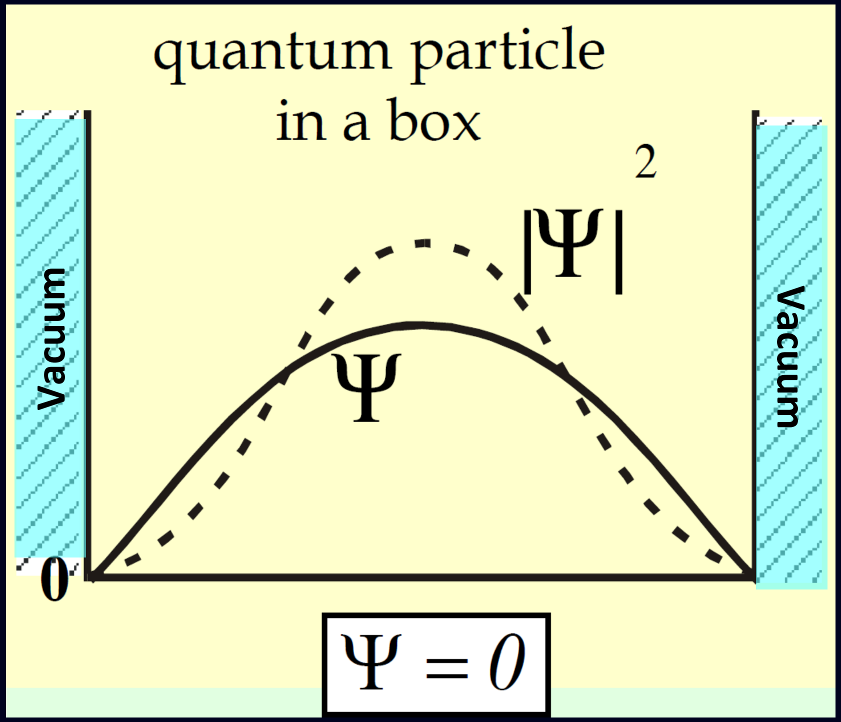

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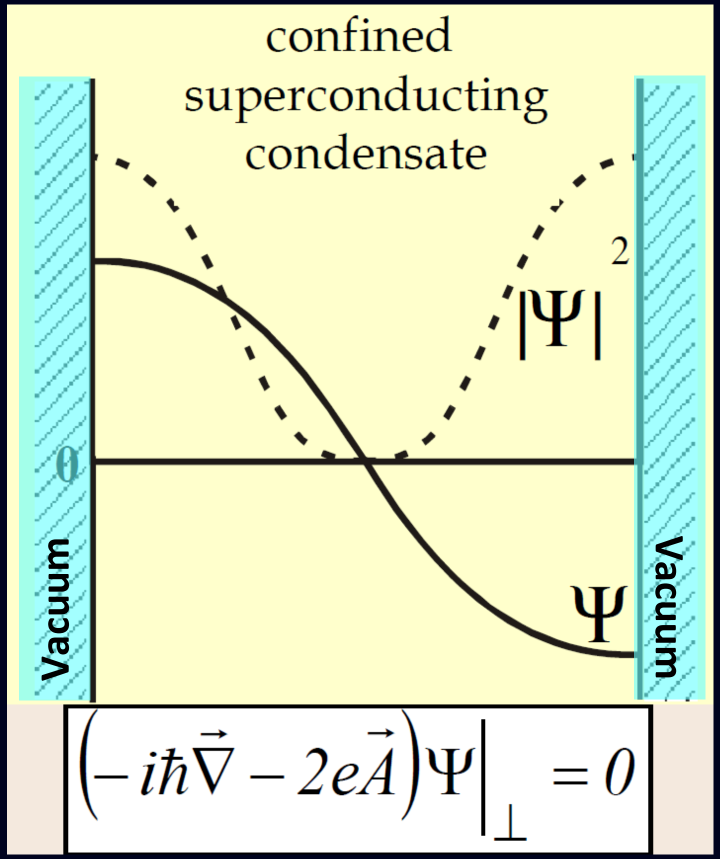
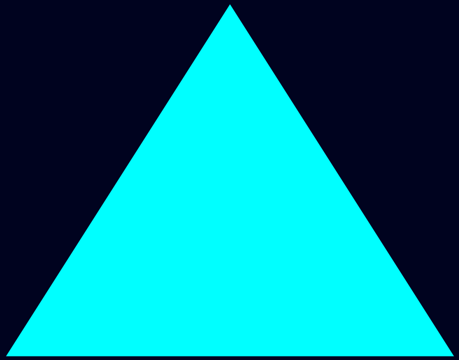
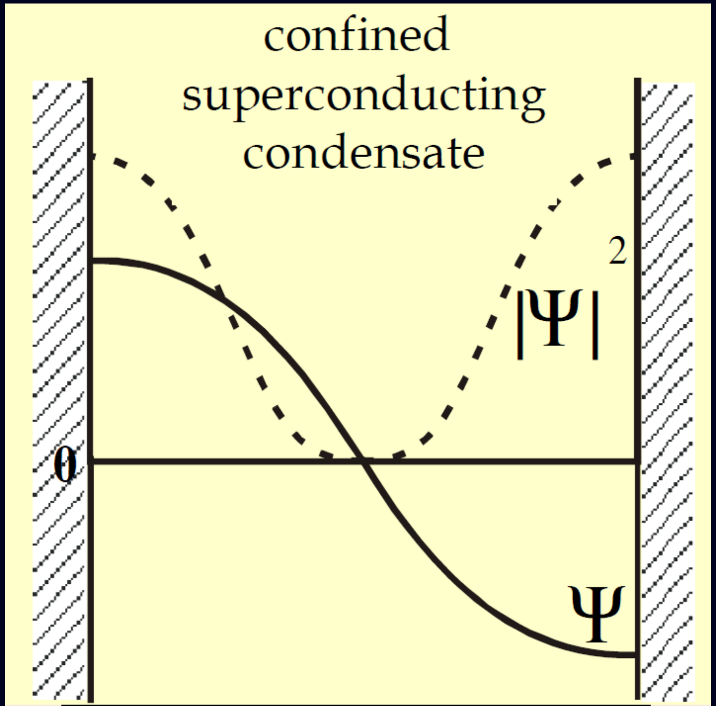


The normal component of the gradient of Ψ is zero
(Supercurrent cannot flow outwards)

Boundary conditions: Confined Superconductors

Where would the order parameter nucleate first?





$$\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)\Psi\Big|_{\perp}=0$$

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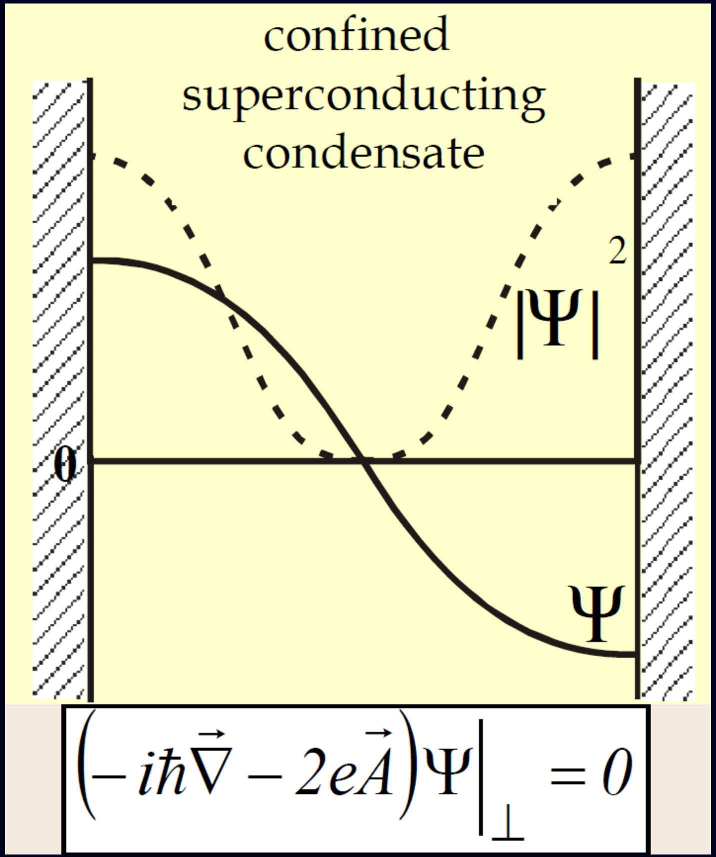

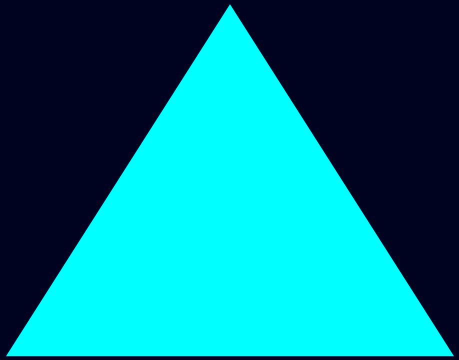

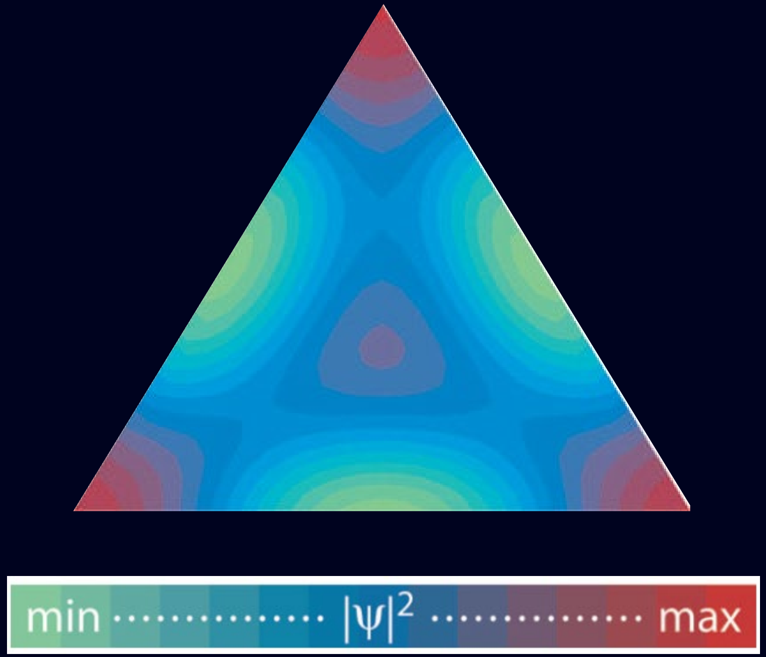

Moshchalkov et al, Phys. Rev. Lett. 86, 7 (2001)

Once in the SC state, only two length scales matter: ξ and λ

$$\Psi = |\Psi(\mathrm{r})|e^{i\varphi(r)}$$

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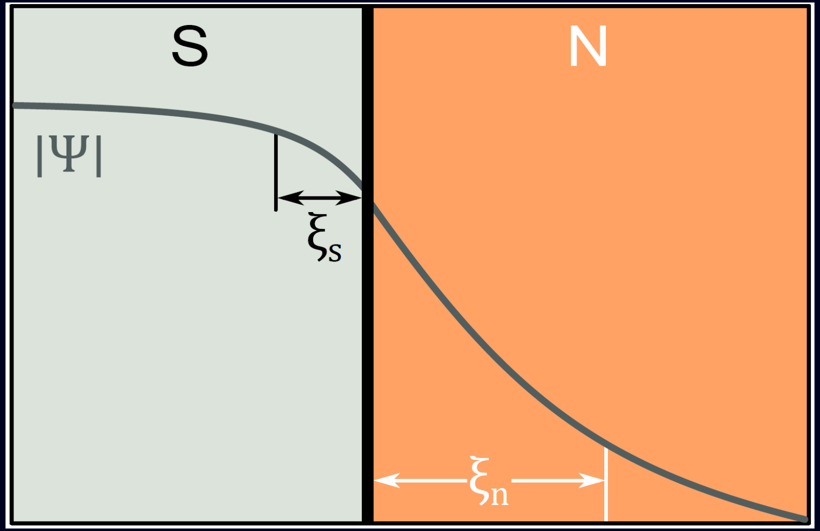
ξ : Coherence Length
"stiffness" of the amplitude

How rapidly does |Ψ| (Cooper pair density) "bend" in real space



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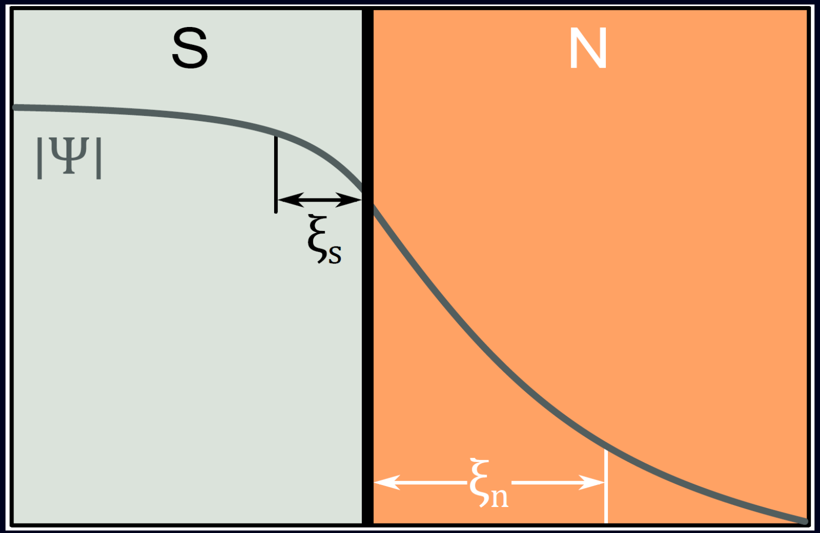
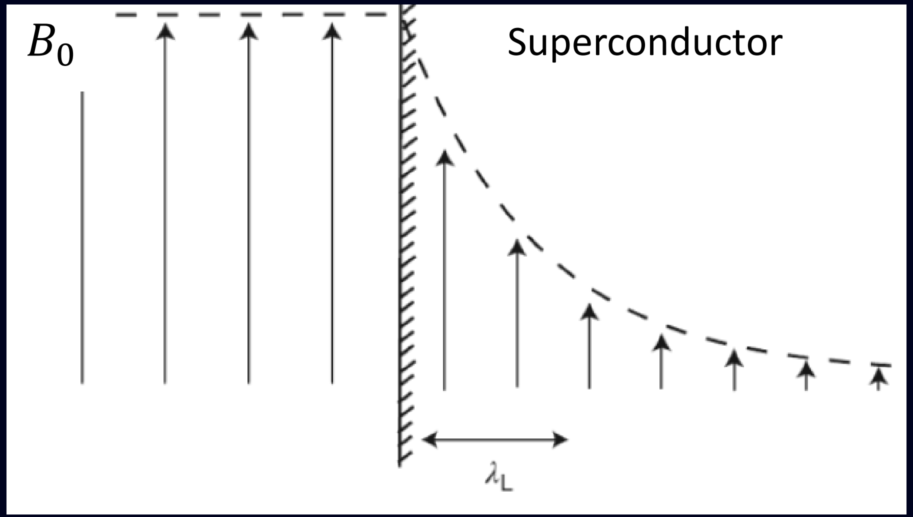
How rapidly does |Ψ| (Cooper pair density) "bend" in real space

λ: magnetic penetration depth
(stiffness of the phase φ)

λ: Characteristic decay length of magnetic fields inside a SC

What's the link between φ and screening of magnetic field?





Once in the SC state, only two length scales matter: ξ and λ

$$\Psi = |\Psi(r)|e^{i\varphi(r)}$$

ξ : Coherence Length
"stiffness" of the amplitude

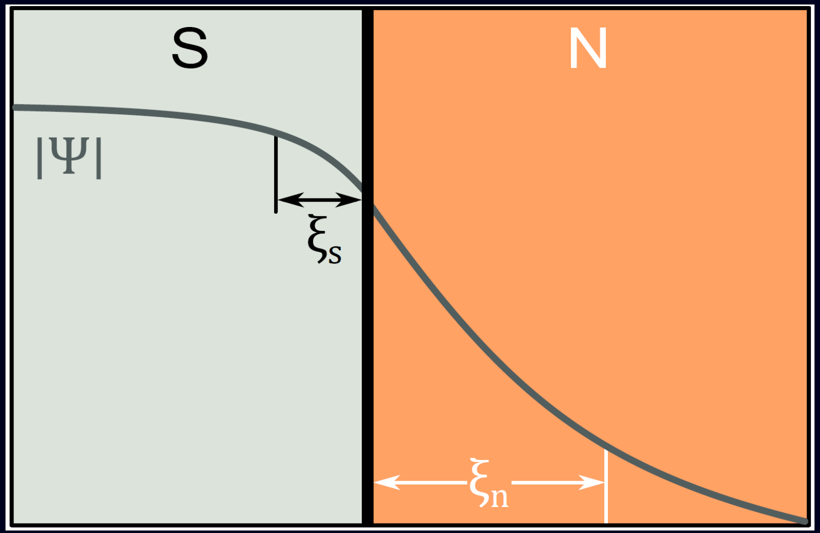
How rapidly does $|\Psi|$ (Cooper pair density) "bend" in real space

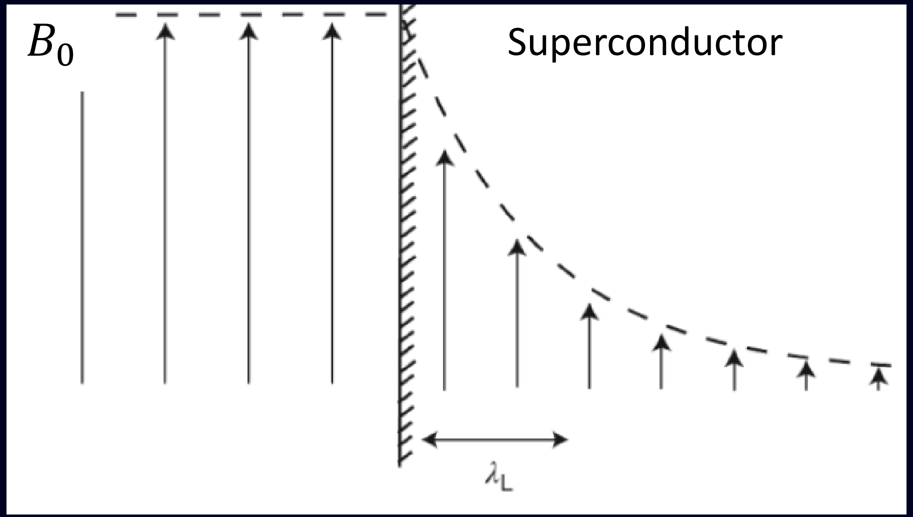
λ: magnetic penetration depth
(stiffness of the phase φ)

λ: Characteristic decay length of magnetic fields inside a SC

What's the link between φ and screening of magnetic field?

λ and ξ are <u>independent</u> material parameters.





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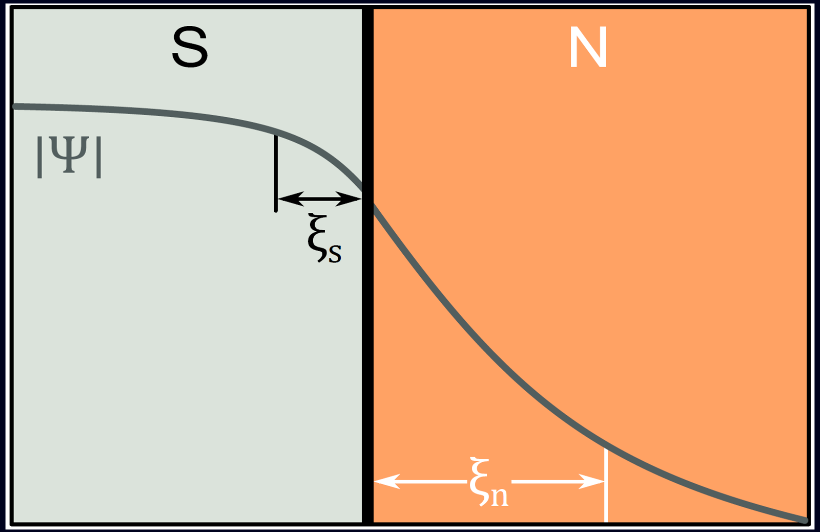
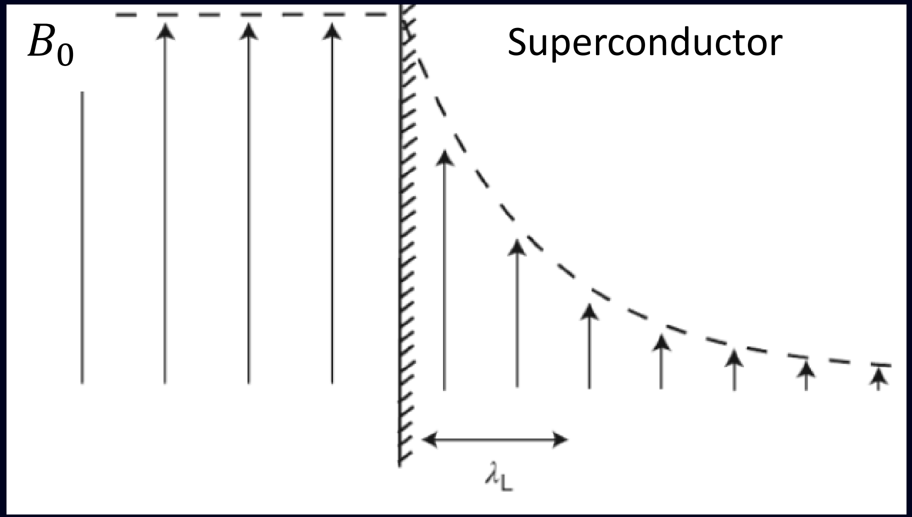
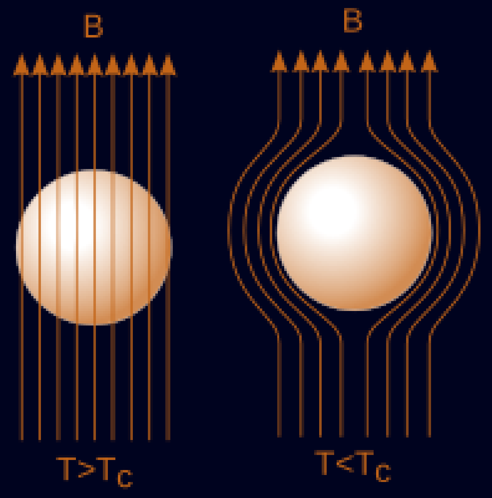
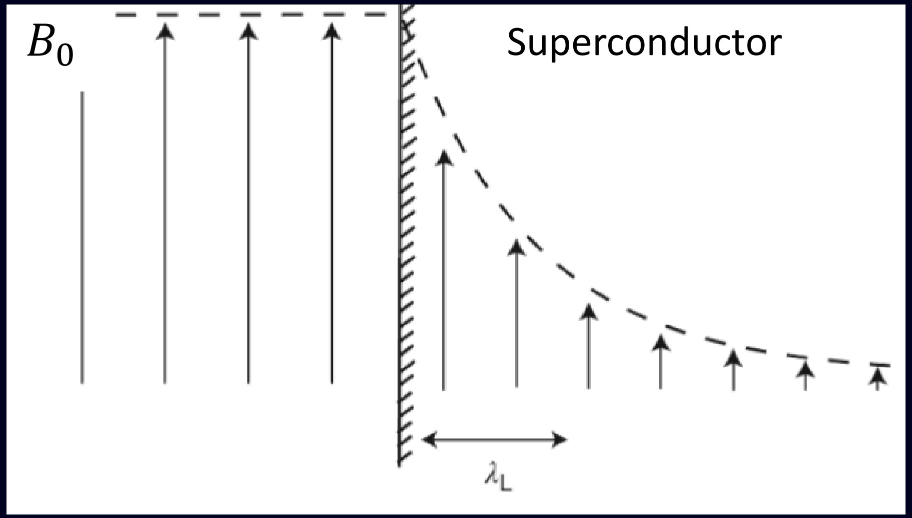




What is the physical significance of λ /ξ?
(see later)

Food for thought:
Do superconductors always expel magnetic field?





$$\Psi = |\Psi(\mathrm{r})| e^{i\varphi(r)}$$

True or false:

When an external magnetic field is applied to a SC, it induces a supercurrent, whose role is to cancel out the applied field.

Self study:
Find out what happens to λ & ξ near T_c

End of Lecture 1

A lot of the material covered in this lecture can be found on Moshchalkov's book

Kaveh Lahabi (2025)