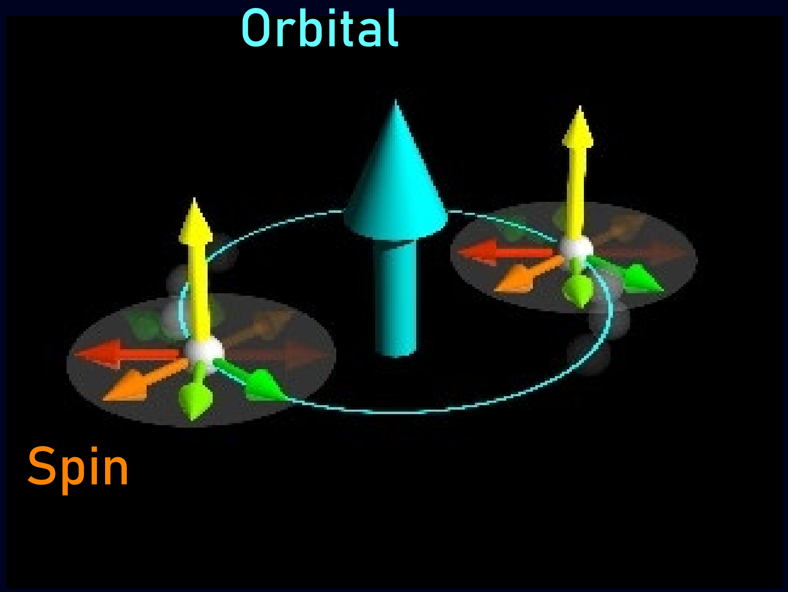
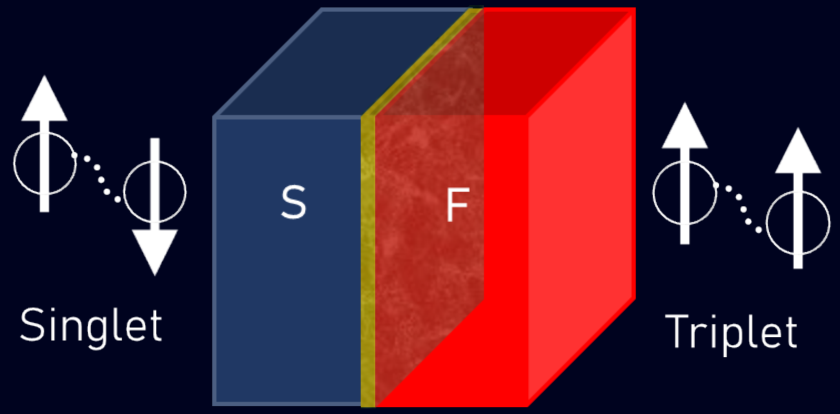
Superconductivity Lecture 4: Symmetries (continued)

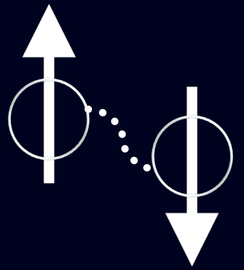




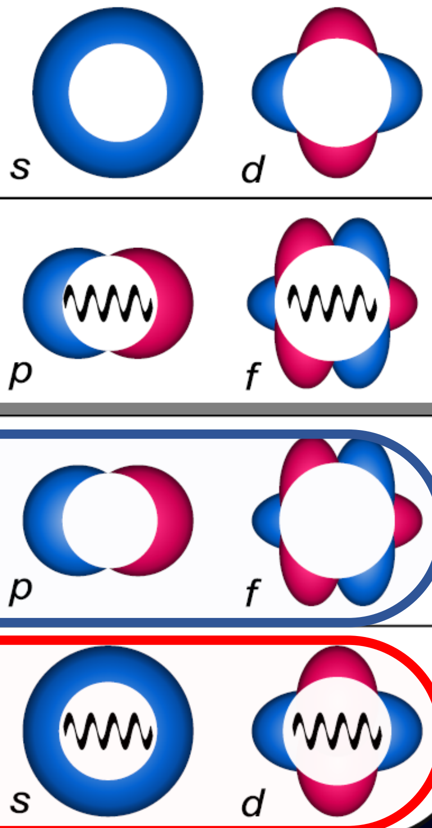
Kaveh Lahabi (2025)

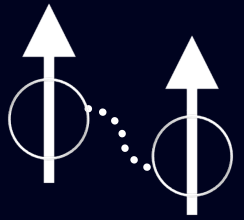
Allowed pairing symmetries (recap)

$$\Psi\,(\sigma_{1,2}\,;\,k_{1,2}\,;\,\omega_{1,2})$$



Spin	Frequency	Momentum	
Singlet (odd) ↑↓ − ↓↑	Even	Even	*s*, *d*
	Odd	Odd	*p*, *f*
Triplet (even) ↑↓ + ↓↑, ↑↑, ↓↓	Even	Odd	*p*, *f*
	Odd	Even	*s*, *d*





S-wave: Nb, Al, MoGe,…
d-wave: Cuprates (e.g. YBCO)

^{3}He, UPt_3

So far, only observed in S-F hybrids (via proximity effect)

Time-reversal symmetry breaking (TRSB)

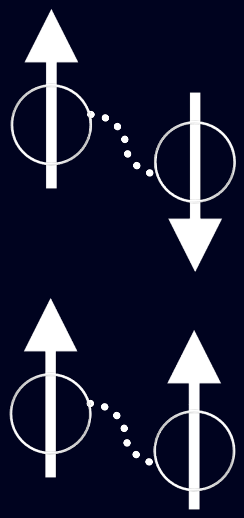
Spontaneous Time-reversal symmetry breaking (TRSB)

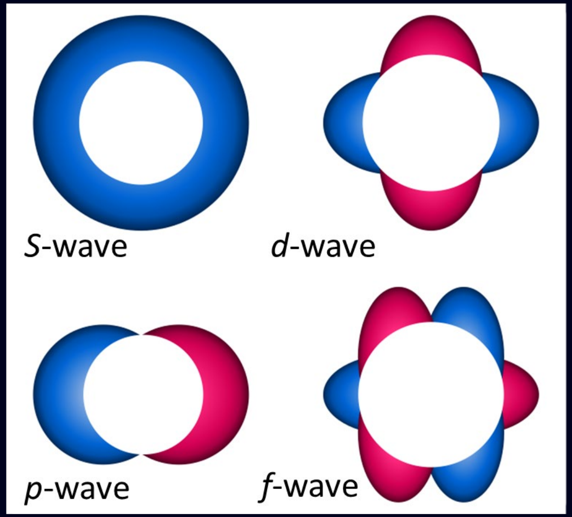
What does it mean?

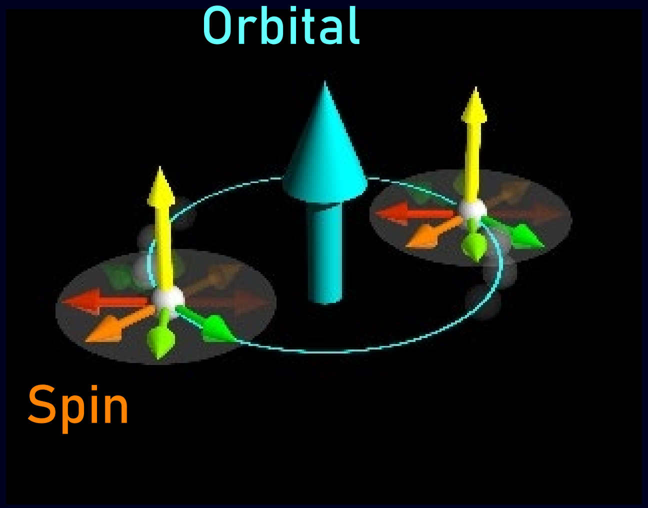
Examples in nature?

Can TRSB happen in superconductors?

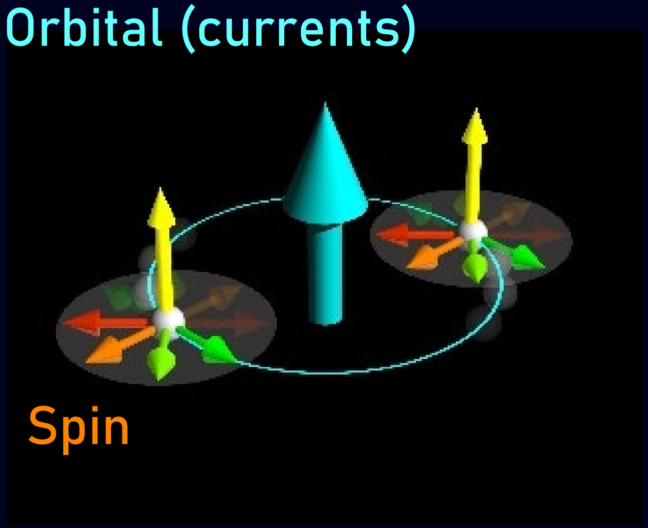
What pairing symmetry shows TRSB?







Detecting spontaneous TRSB in SCs (with no applied field)



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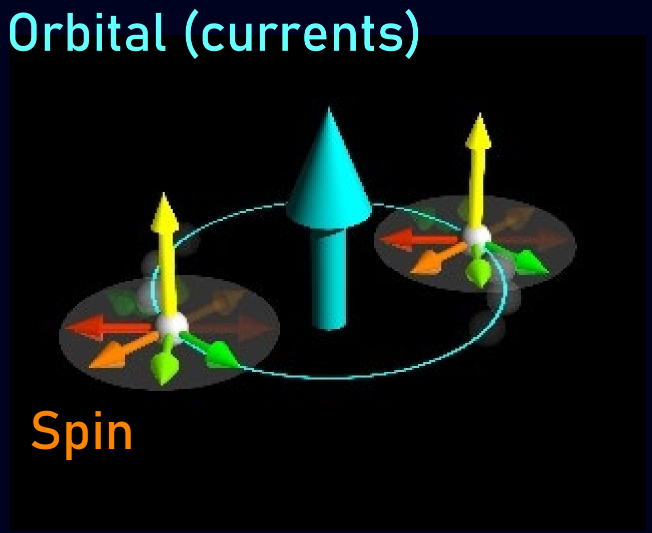
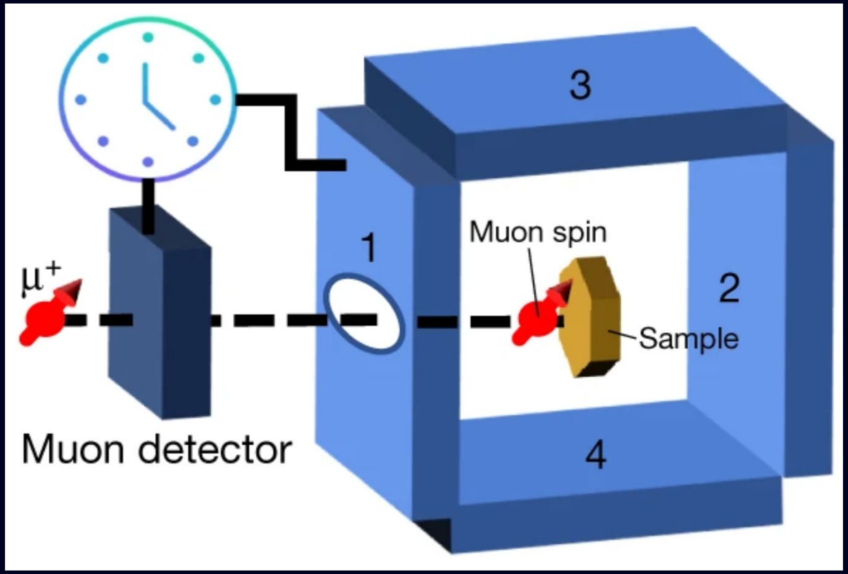


zero-field μSR



C. Mielke et al Nature (2022)

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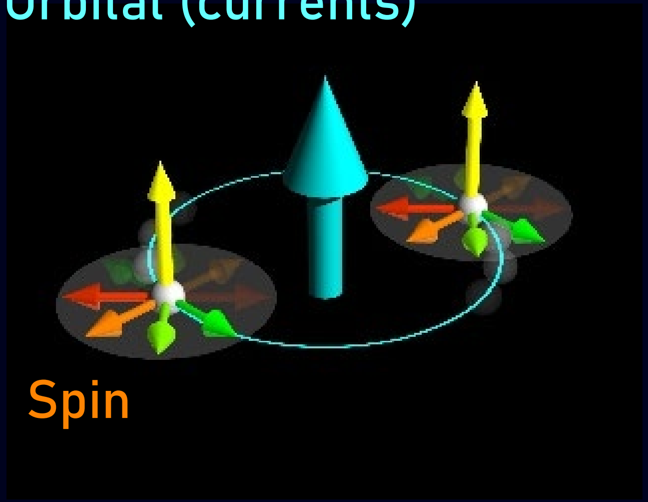

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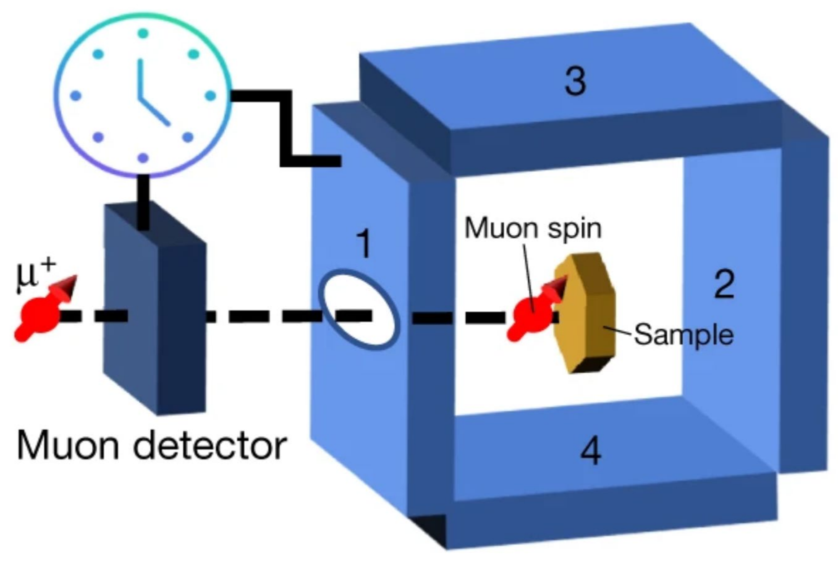
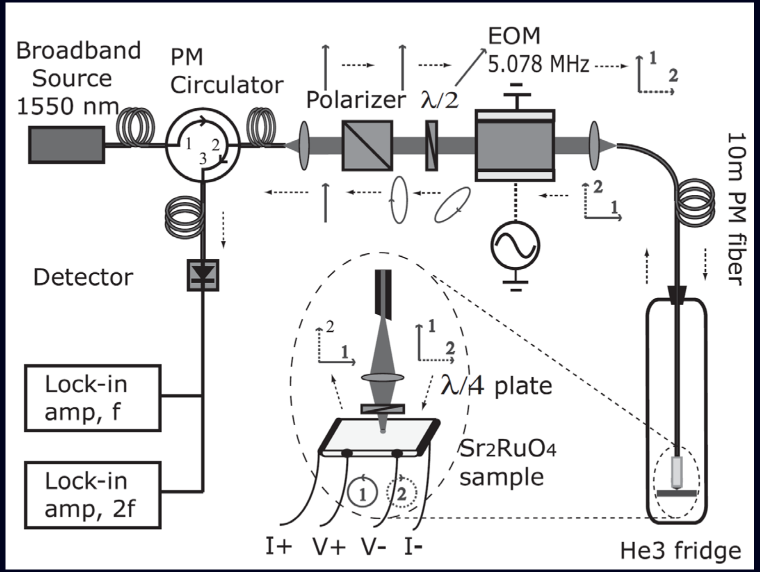


C. Mielke et al Nature (2022)

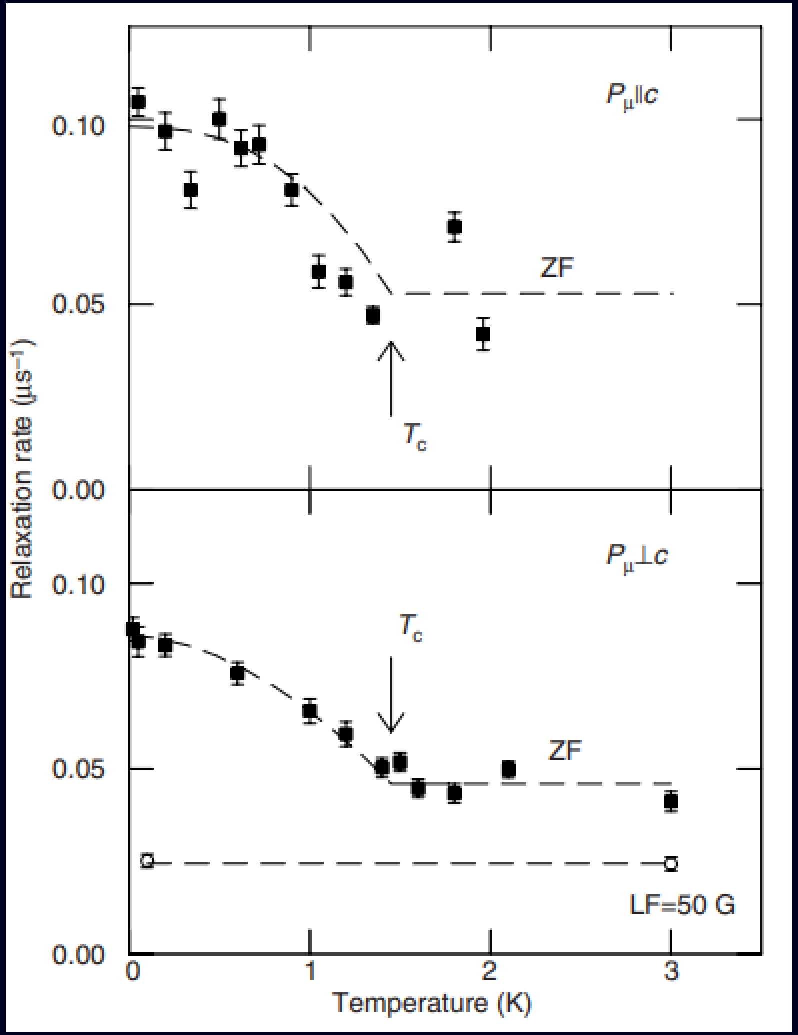
High-Resolution Polar Kerr



Kapitulnik et al PRL (2003)

Famous example: Spontaneous TRSB in Sr_2RuO_4

Muon spin resonance (Luke et al, nature, 1998)



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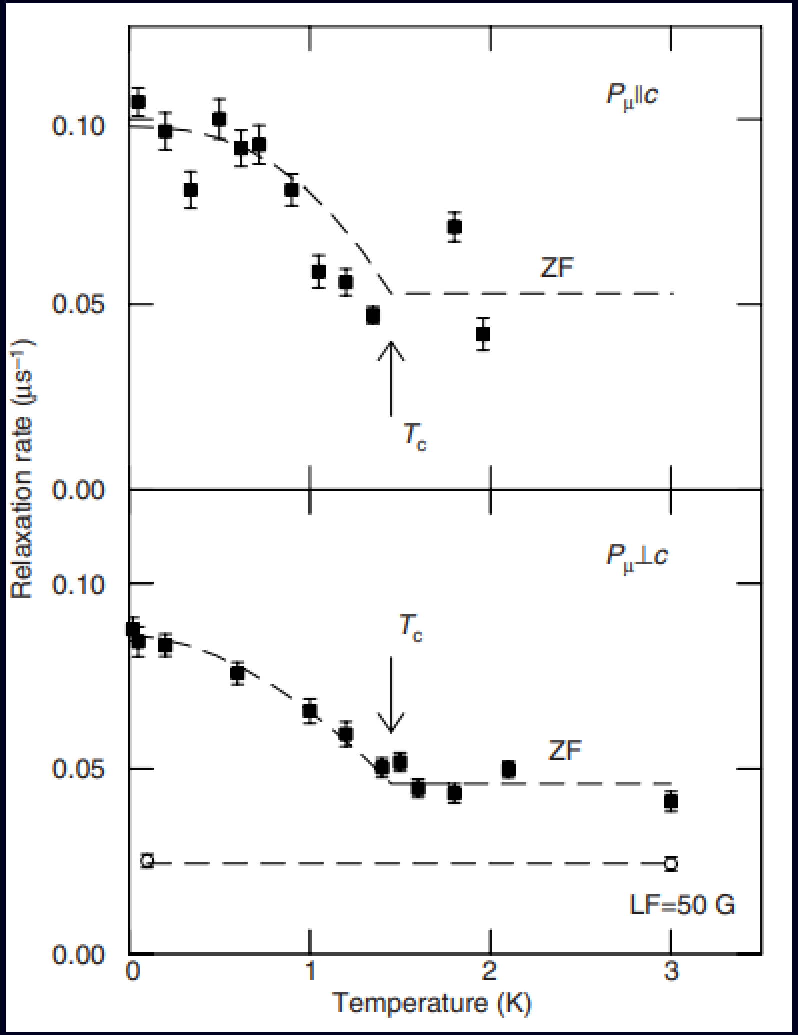
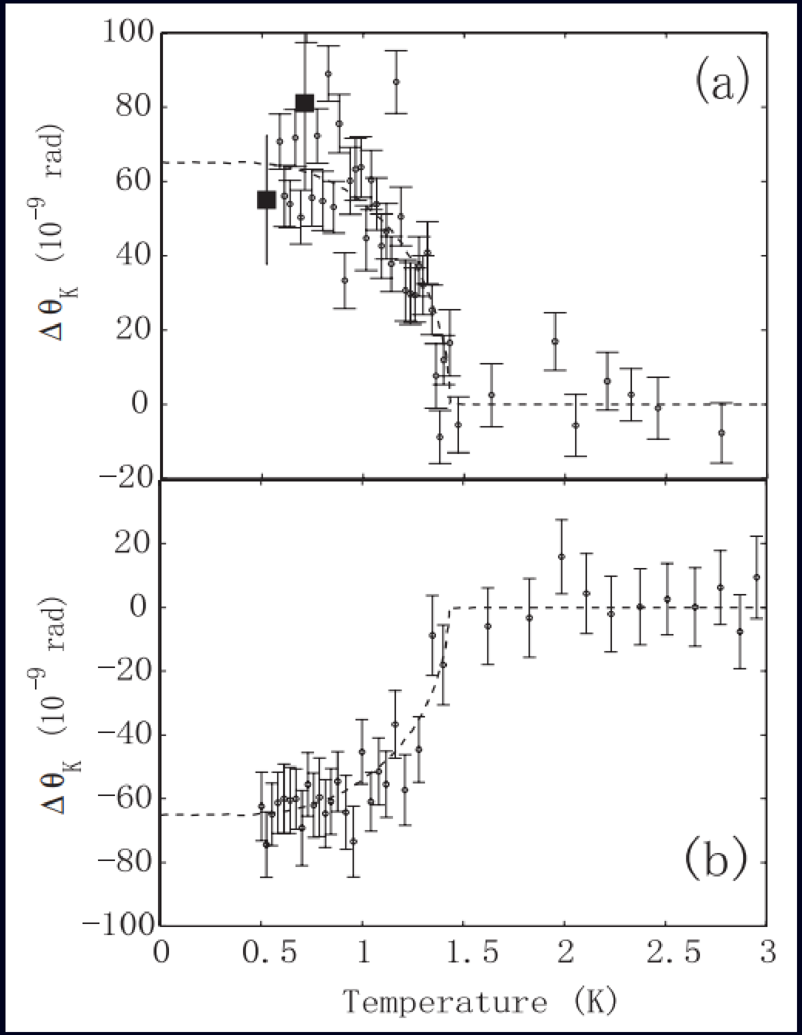


Polar Kerr effect (Xia et al, PRL, 2006)



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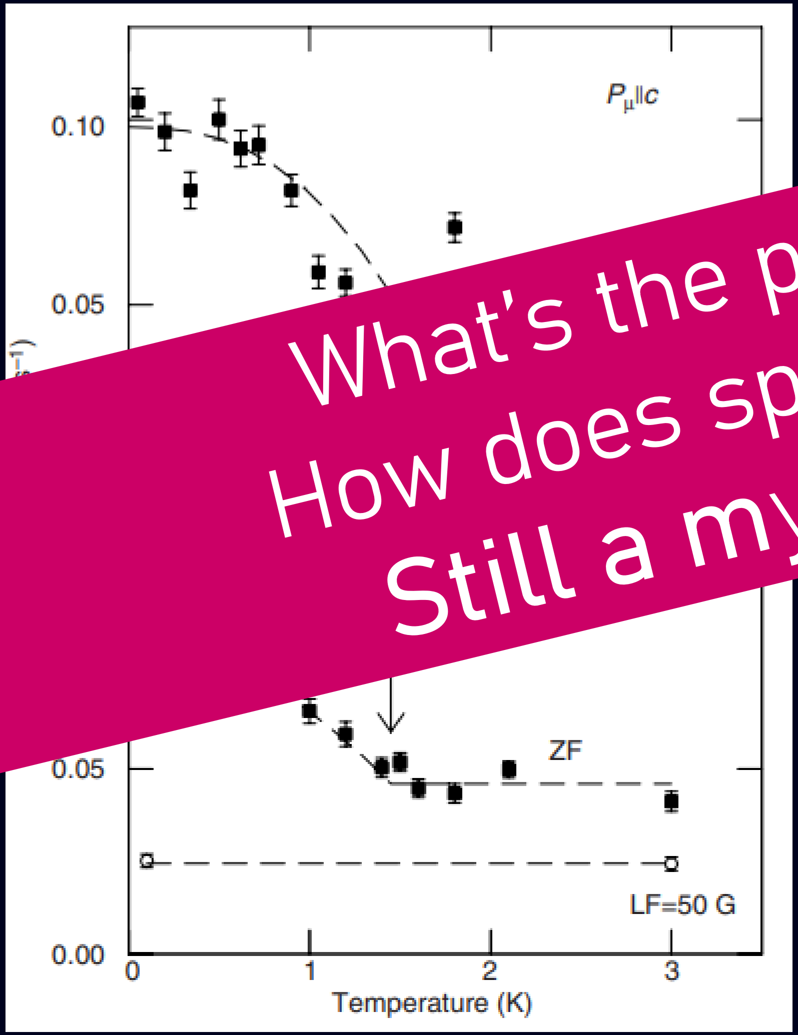

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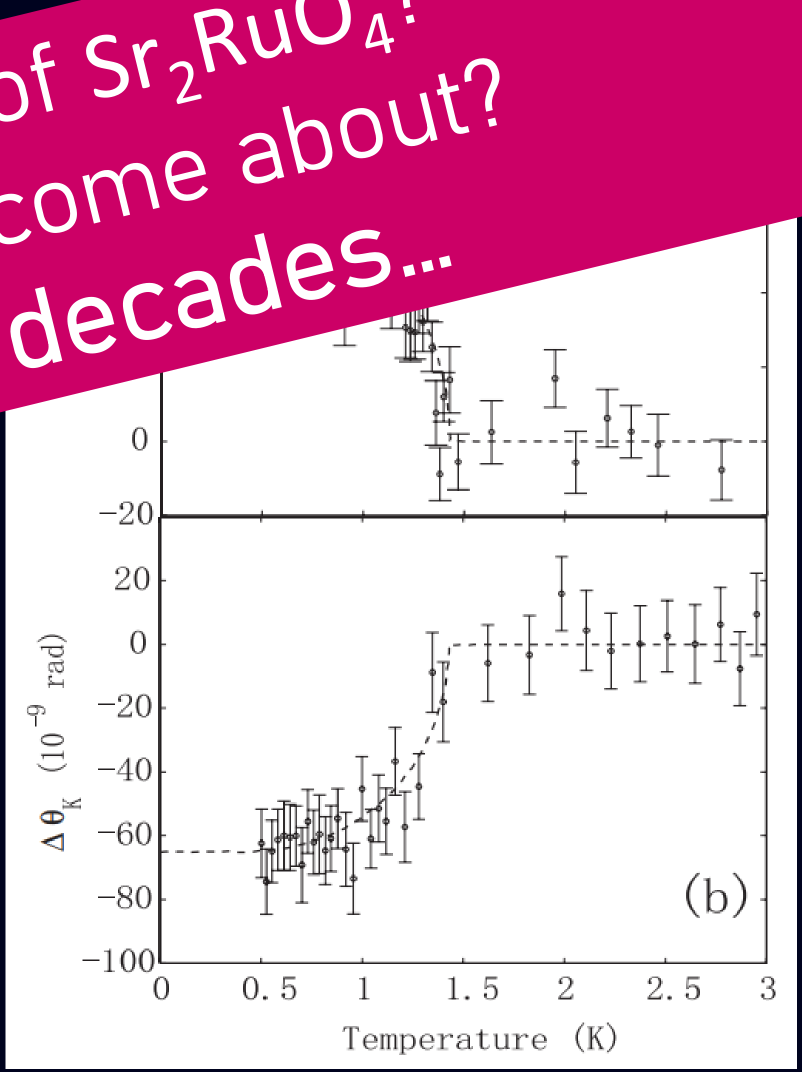


What's the pairing symmetry of Sr_2RuO_4?

How does spontaneous TRSB come about?

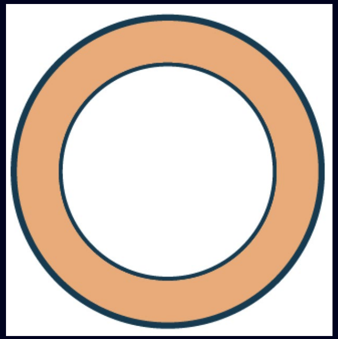
Still a mystery to after 3 decades…

What kind of pairing symmetry results in spontaneous TRSB?

Understanding symmetries by visualizing them

Singlet states

S-wave



L = 0
σ = 0
Δ is constant

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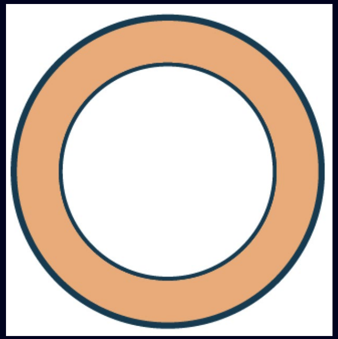
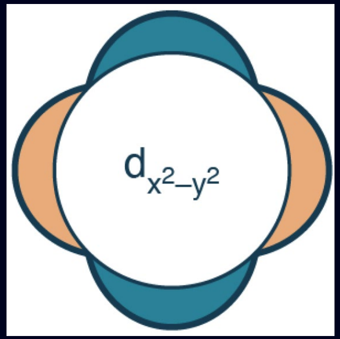
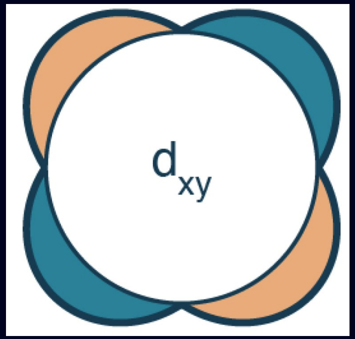


d-wave





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Δ is k-dependent

Magnitude and phase of Δ varies in (real and k) space

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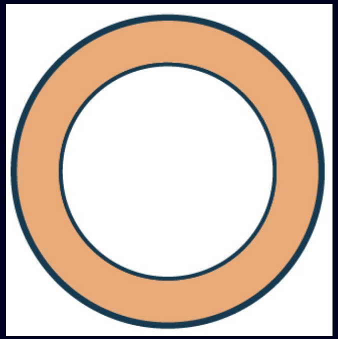

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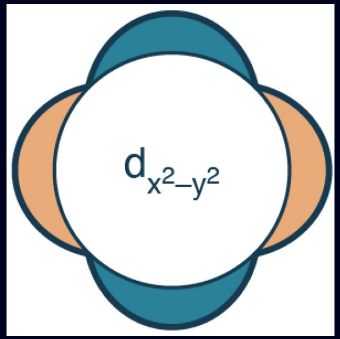

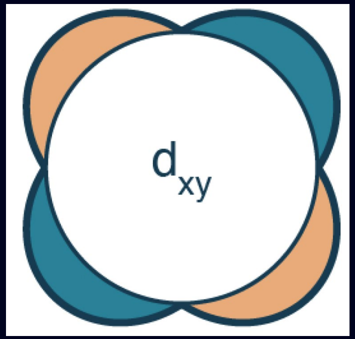

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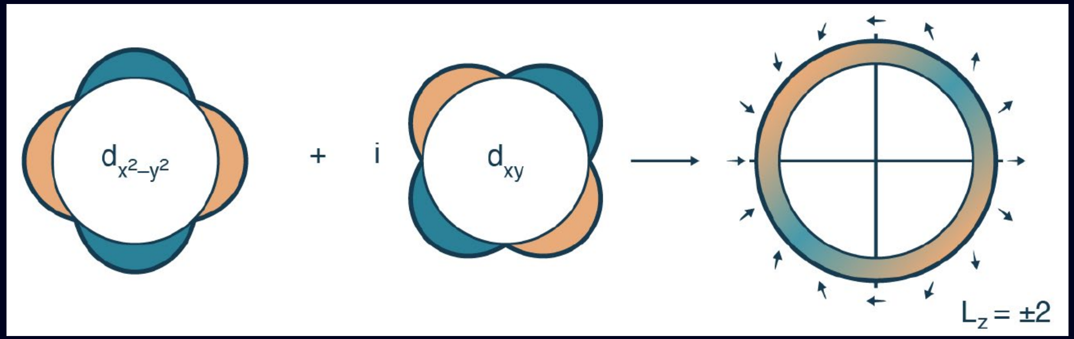
Hybrid gaps (e.g., Chiral d-wave)



Gap can be isotropic, despite being d-wave

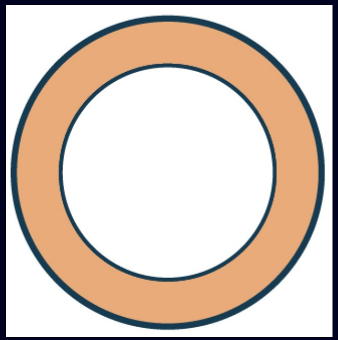
Can have a net orbital angular momentum

Chirality → phase winding has a directions

Understanding symmetries by visualizing them

Singlet states

S-wave



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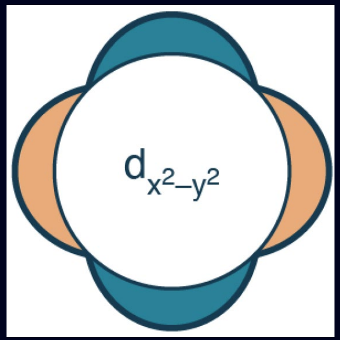

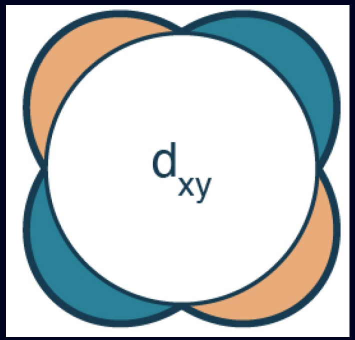

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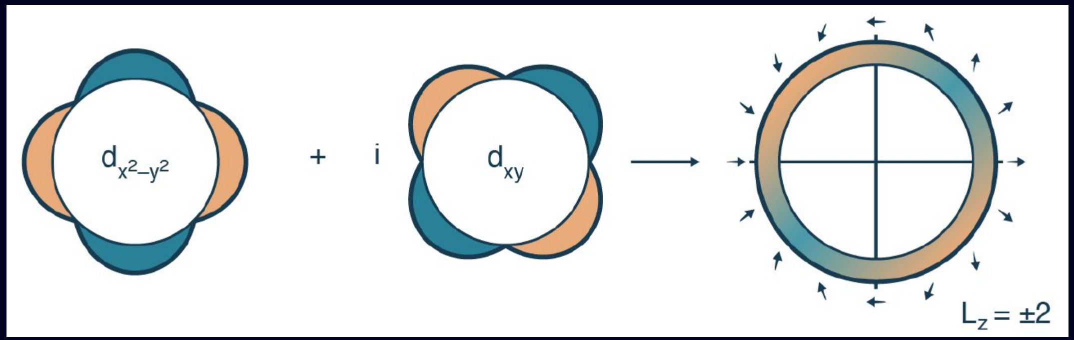

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Triplet states:

3 independent vectors describe the spin symmetry of $\Delta(\mathrm{k})$

$$\Delta(\boldsymbol{k}) = \begin{pmatrix} \Delta_{\boldsymbol{k},\uparrow\uparrow} & \Delta_{\boldsymbol{k},0} \\ \Delta_{\boldsymbol{k},0} & \Delta_{\boldsymbol{k},\downarrow\downarrow} \end{pmatrix}$$

$\Delta_{\uparrow\uparrow}$ $|\uparrow\uparrow\rangle$ $m_z = +1$

$\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow} = \Delta_0$ $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ $m_z = 0$ z

$\Delta_{\downarrow\downarrow}$ $|\downarrow\downarrow\rangle$ $m_z = -1$

The elegance of *d*-vector formalism: One vector to rule them all

For a given quantization direction, $\Delta_{\uparrow\uparrow}$ and $\Delta_{\downarrow\downarrow}$ represent spin projections of +1 and −1, respectively, while $\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow} = \Delta_0$ corresponds to triplet pairing with zero spin projection (i.e. Cooper pairs do have a spin $S = 1$, but it lies perpendicular to the quantization axis). This gap matrix can be elegantly reduced to a three-dimensional complex vector $\boldsymbol{d}(\boldsymbol{k}) = [d_x(\boldsymbol{k}), d_y(\boldsymbol{k}), d_z(\boldsymbol{k})]$ (known as the d-vector), defined by

$$\begin{pmatrix} \Delta_{\boldsymbol{k},\uparrow\uparrow} & \Delta_{\boldsymbol{k},0} \\ \Delta_{\boldsymbol{k},0} & \Delta_{\boldsymbol{k},\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} -d_x(\boldsymbol{k}) + i d_y(\boldsymbol{k}) & d_z(\boldsymbol{k}) \\ d_z(\boldsymbol{k}) & d_x(\boldsymbol{k}) + i d_y(\boldsymbol{k}) \end{pmatrix}$$

A state is called unitary if $|\boldsymbol{d}(\boldsymbol{k}) \times \boldsymbol{d}^*(\boldsymbol{k})| = 0$. In this case, $\boldsymbol{d}(\boldsymbol{k})$ has a straightforward meaning: its amplitude is proportional to size of the gap at $(\boldsymbol{k}, -\boldsymbol{k})$; and its direction is perpendicular to the plane of equal-spin paired electrons, where $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ can be defined with respect to any quantization direction in that plane. For instance,

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$$\Delta_{\uparrow\uparrow z} = \Delta_{\downarrow\downarrow z} = 0$$

d-vector	Δ/Δ_0	direction	TRS
$\hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y$	$\sqrt{k_x^2 + k_y^2}$	$\boldsymbol{d} \parallel ab$	preserved
$\hat{\mathbf{x}}k_y - \hat{\mathbf{y}}k_x$	$\sqrt{k_x^2 + k_y^2}$	$\boldsymbol{d} \parallel ab$	preserved
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$\hat{\mathbf{z}}k_x$	$\lvert k_x \rvert$	$\boldsymbol{d} \parallel c$	preserved
$\hat{\mathbf{z}}(k_x + k_y)$	$\lvert k_x + k_y \rvert$	$\boldsymbol{d} \parallel c$	preserved
$\hat{\mathbf{z}}(k_x \pm i k_y)$	$\sqrt{k_x^2 + k_y^2}$	$\boldsymbol{d} \parallel c$	broken

The (famous) chiral *p*-wave pairing symmetry

$$\boldsymbol{d}(\boldsymbol{k}) = \hat{\mathbf{z}}\Delta_0(k_x \pm ik_y)$$

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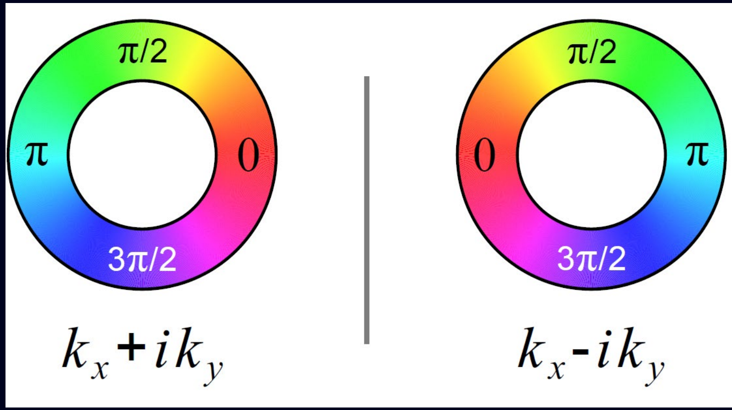

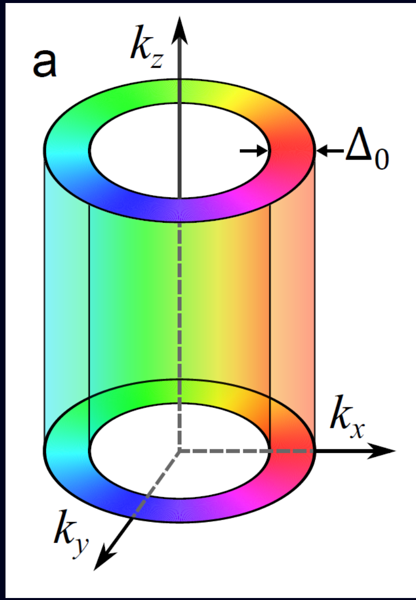

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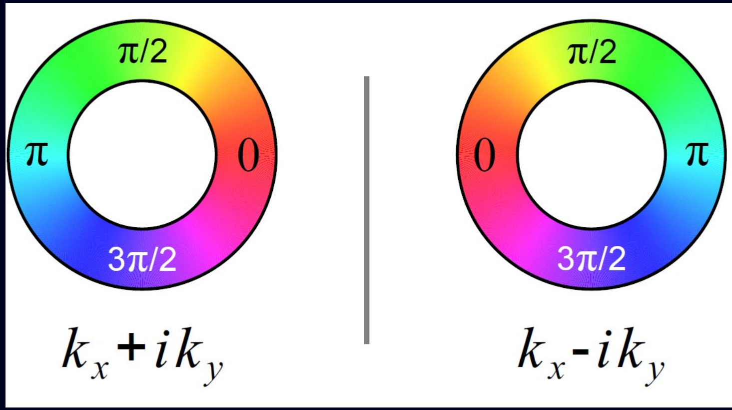
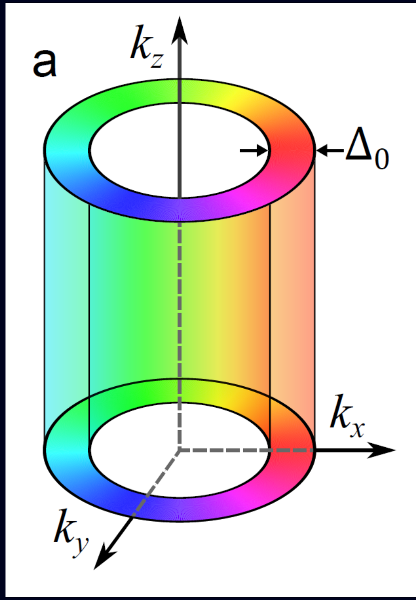
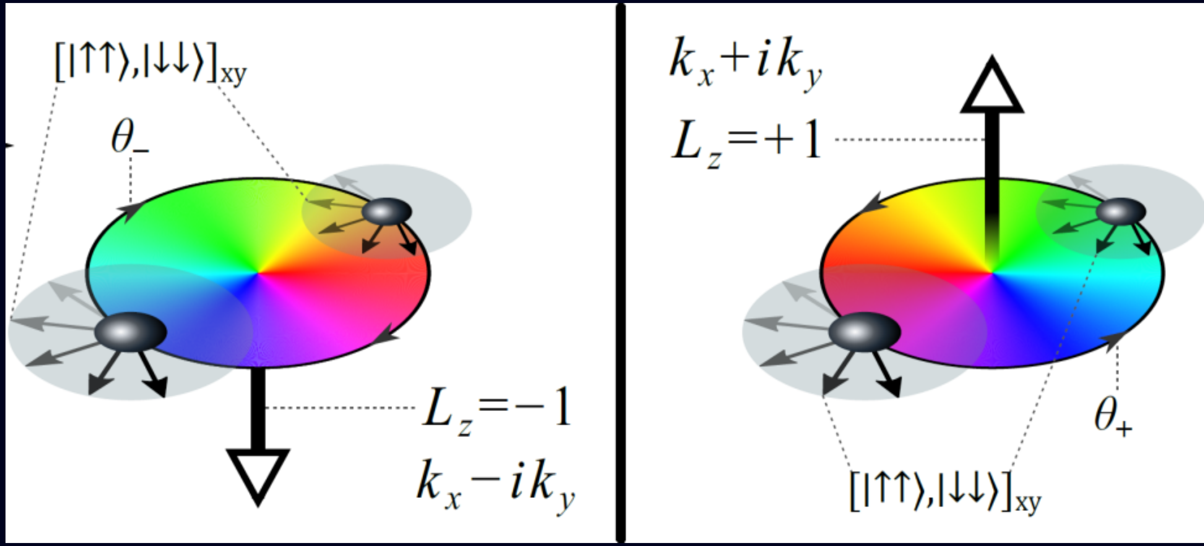


$$\boldsymbol{d}(\boldsymbol{k}) \propto e^{i\theta_k}$$





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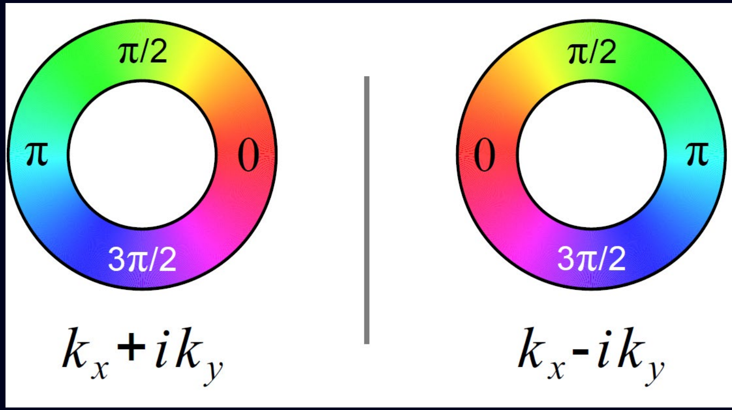
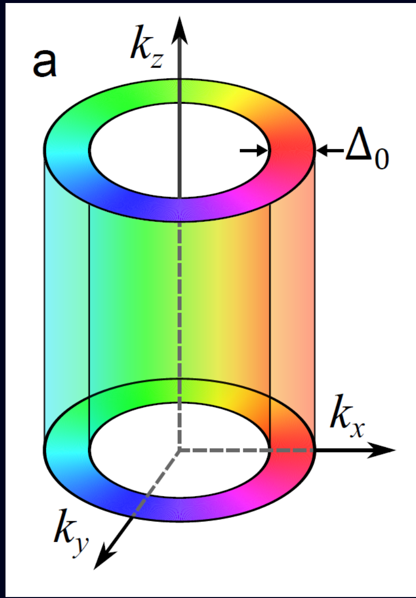
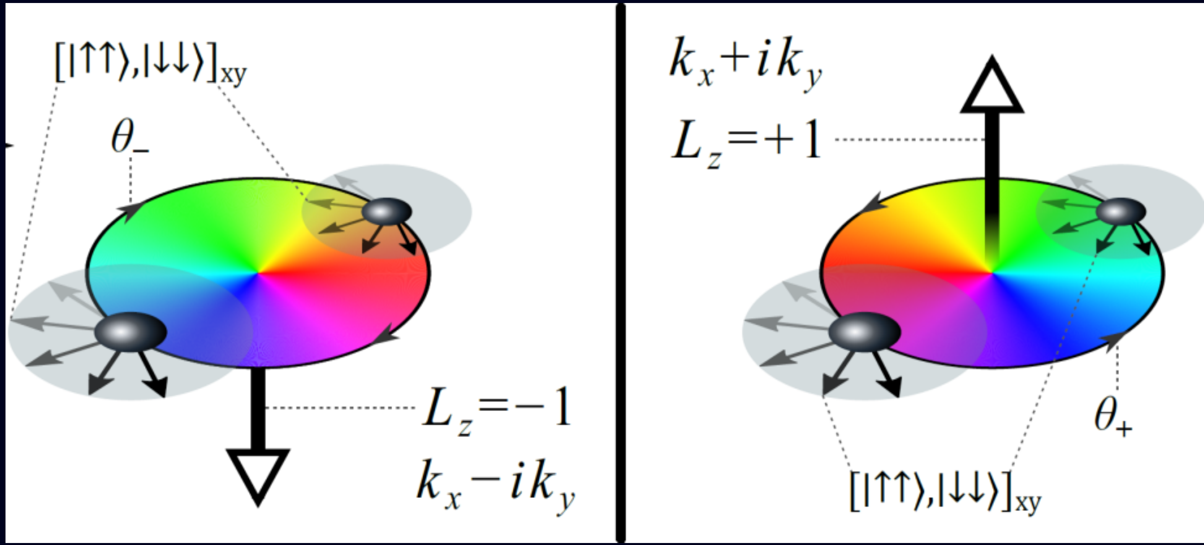
The (famous) chiral *p*-wave pairing symmetry

$$\boldsymbol{d}(\boldsymbol{k}) = \hat{\mathbf{z}}\Delta_0(k_x \pm ik_y)$$





$$\boldsymbol{d}(\boldsymbol{k}) \propto e^{i\theta_k}$$



What's causing the TRSB?
(spin or orbital angular momentum?)

d-vector	Δ/Δ_0	direction	TRS
$\hat{\mathbf{x}}k_x + \hat{\mathbf{y}}k_y$	$\sqrt{k_x^2 + k_y^2}$	$\mathbf{d} \parallel ab$	preserved
$\hat{\mathbf{x}}k_y - \hat{\mathbf{y}}k_x$	$\sqrt{k_x^2 + k_y^2}$	$\mathbf{d} \parallel ab$	preserved
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Why don't the other states break TRS?

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See Mackenzie, Maeno, REVIEWS OF MODERN PHYSICS 75 (2003), or Kaveh's PhD thesis for more examples

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What's happening to the Cooper pair spin?
(Is there a net spin to break TRS?)

What about the orbital part?
(is there a net L?)

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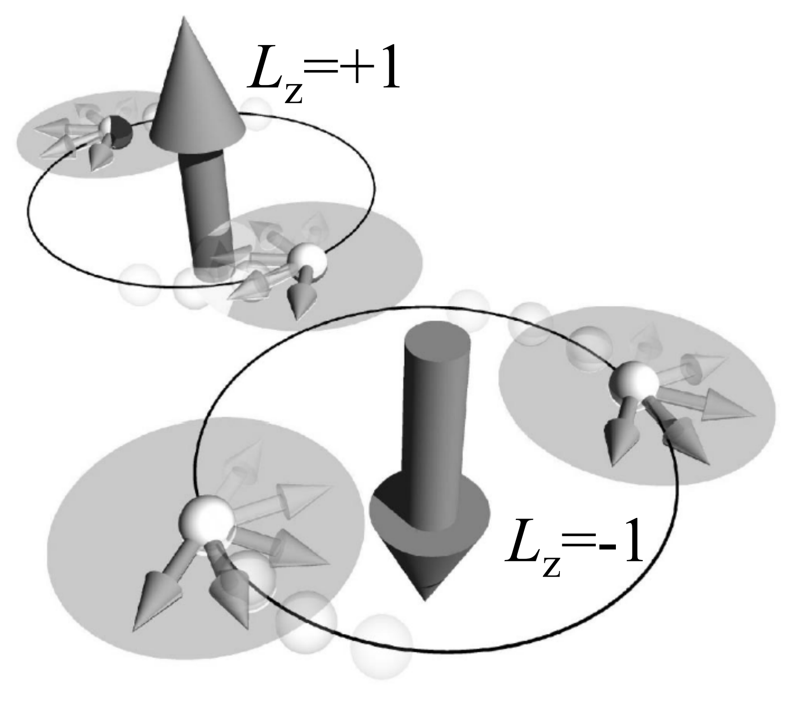

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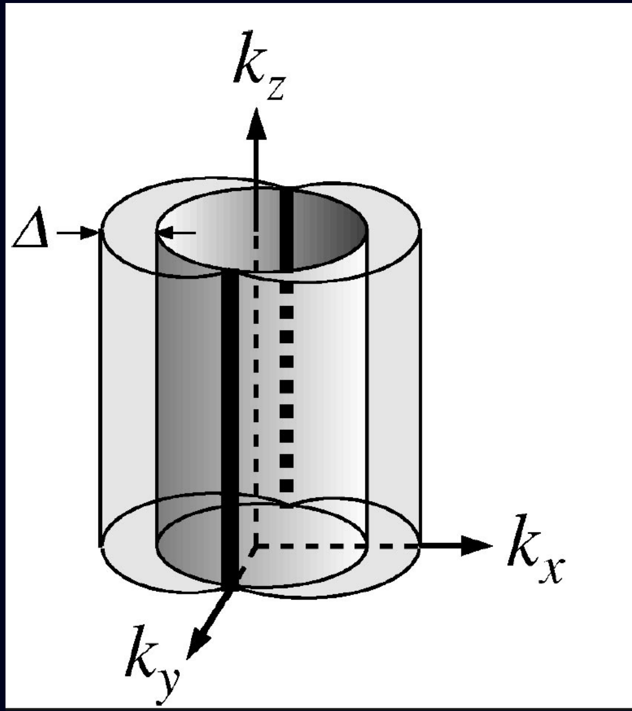

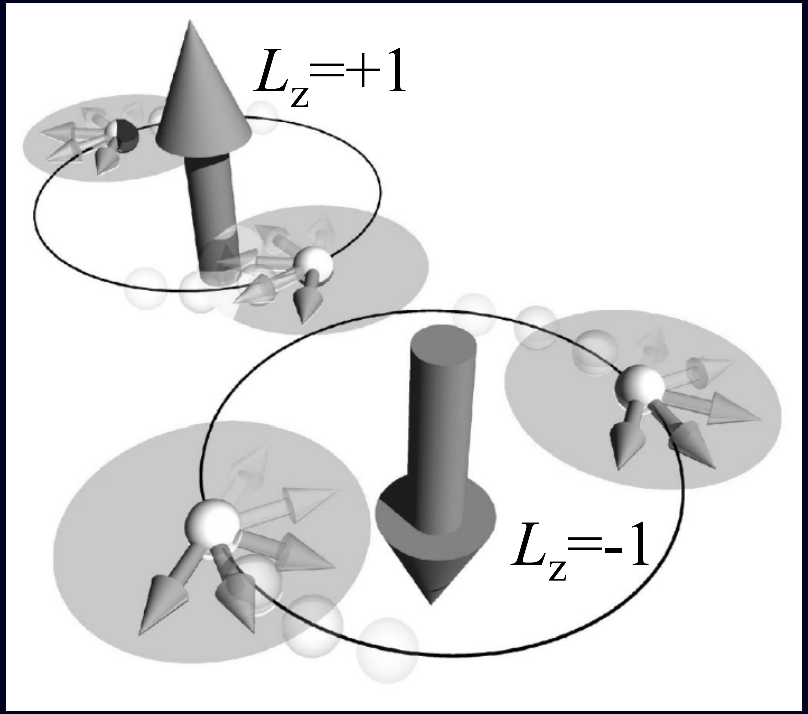

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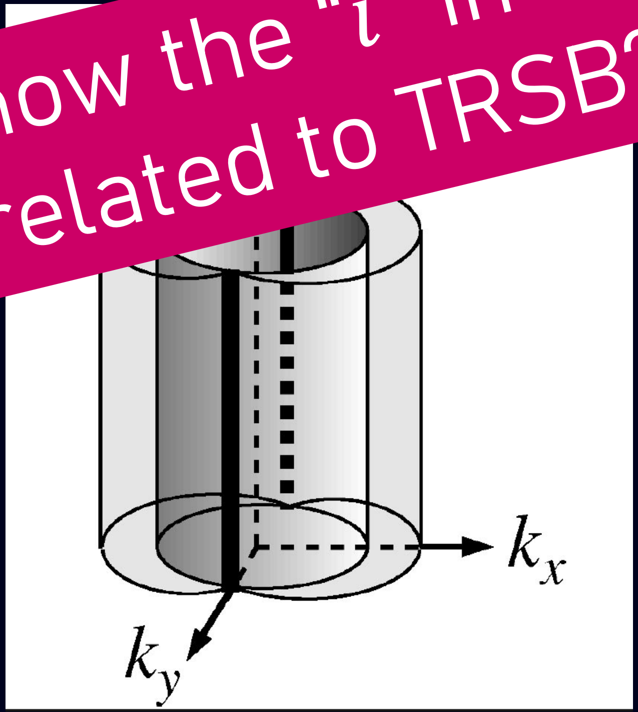
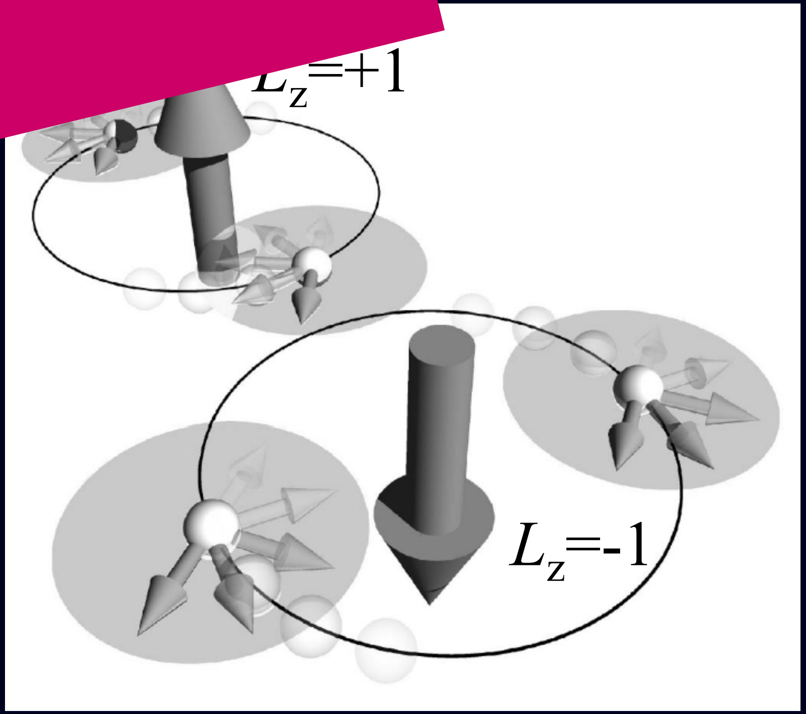
Do you now see how the "*i*" in the chiral states is related to TRSB?





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More examples: TRSB or no TRSB?

$$\mathbf{d} = \Delta_0/2(\hat{\mathbf{x}} + i\hat{\mathbf{y}})(k_x + ik_y)$$

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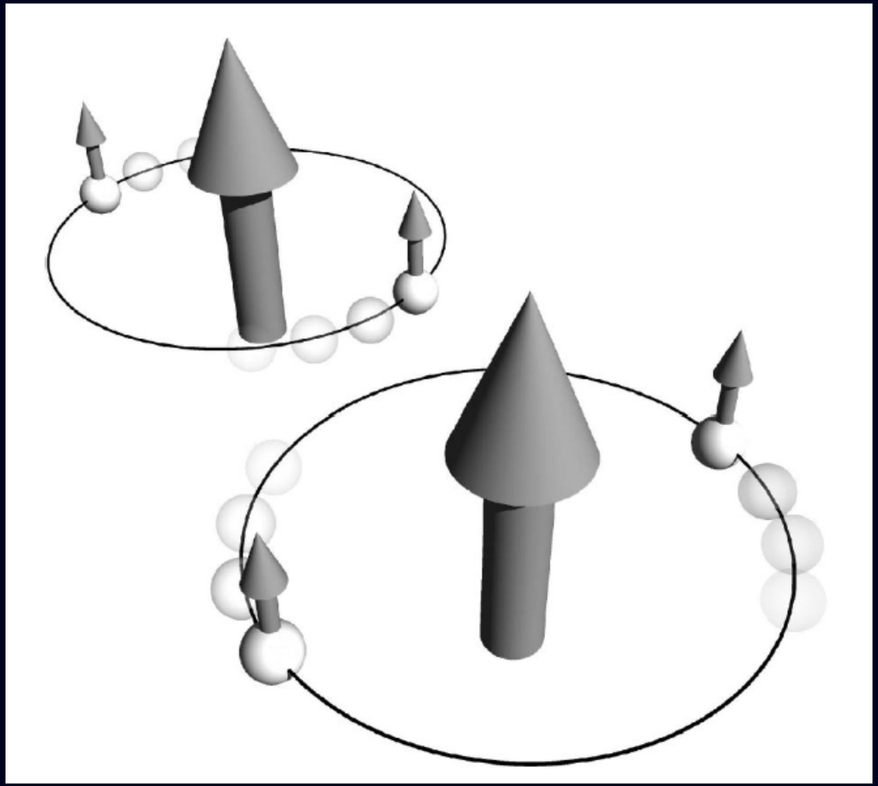

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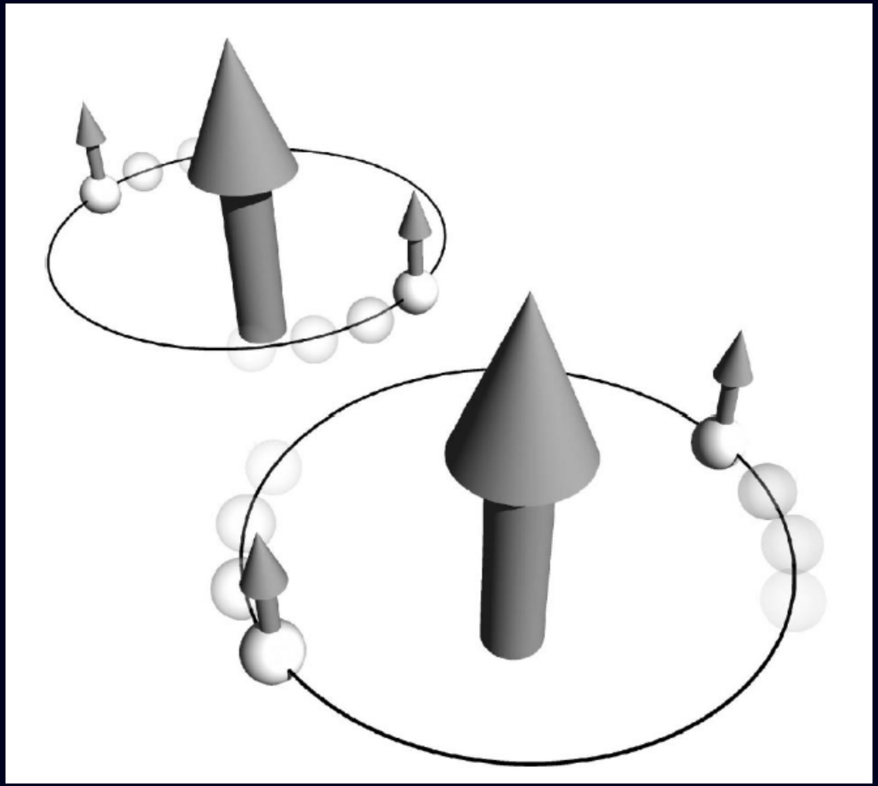

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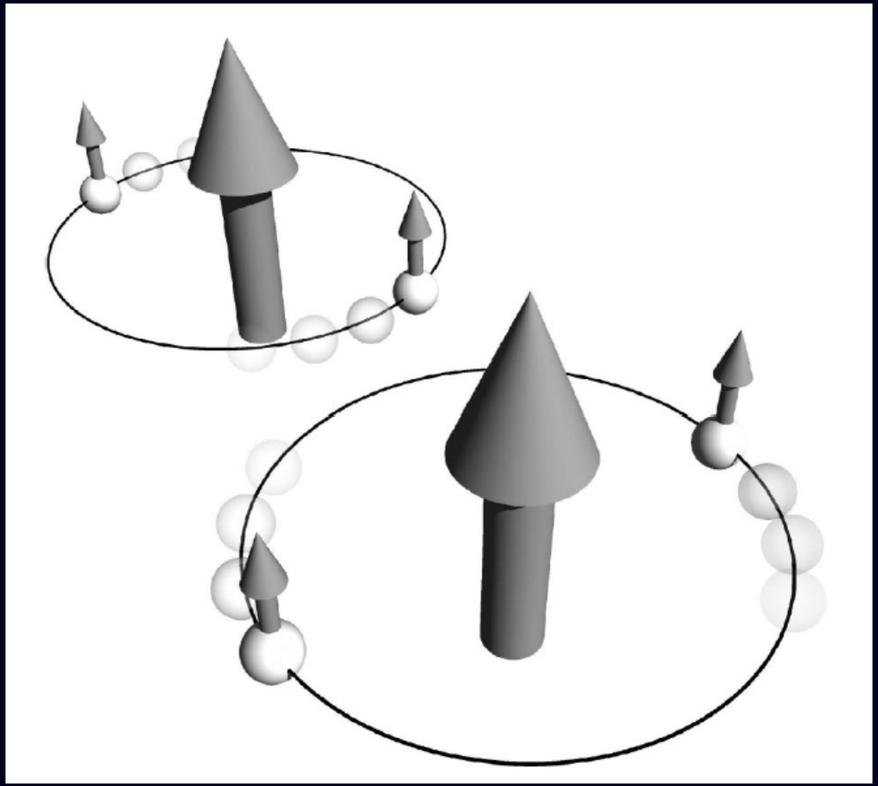

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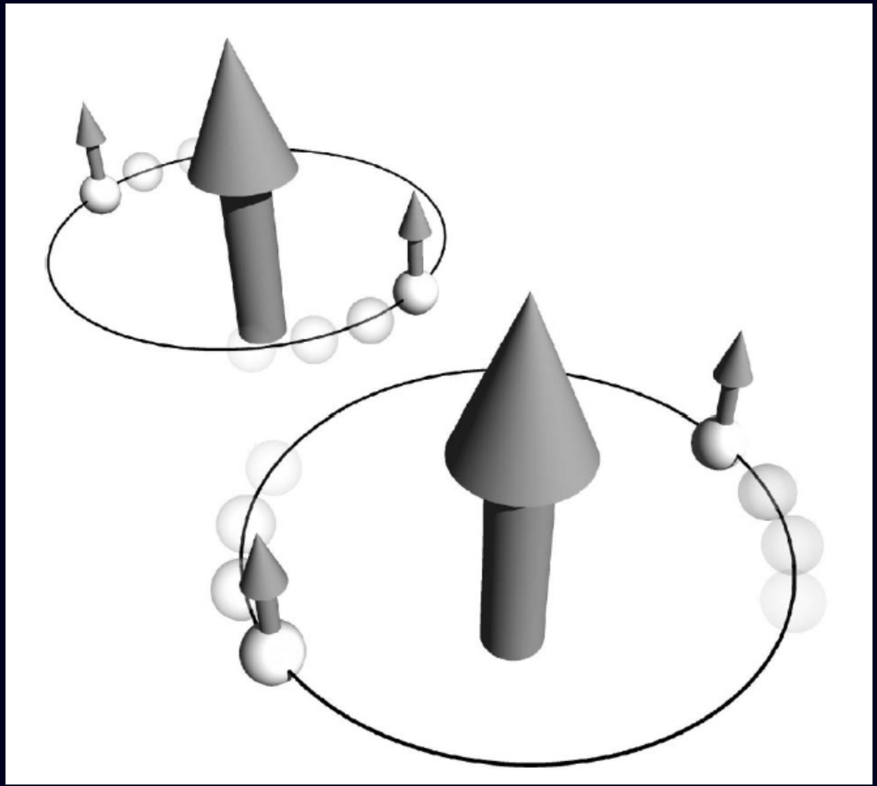

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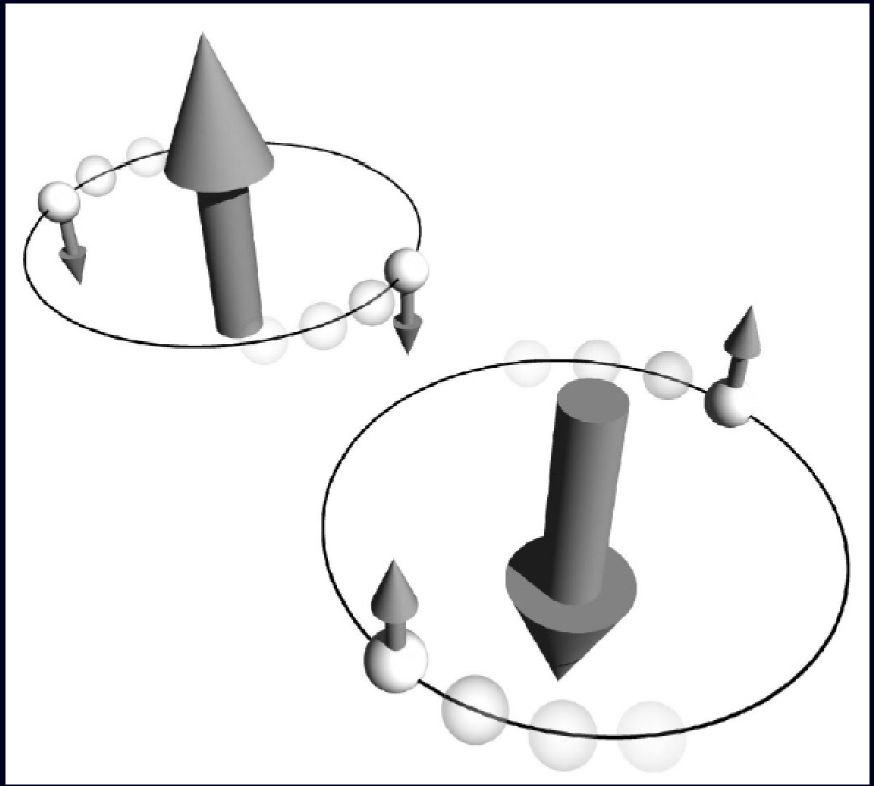
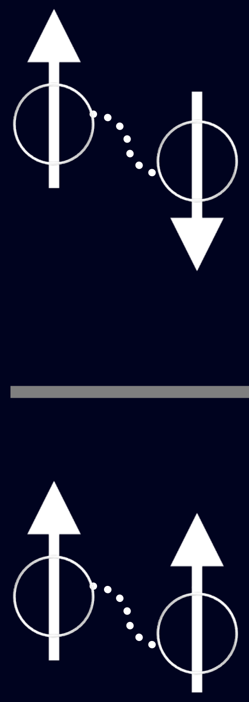
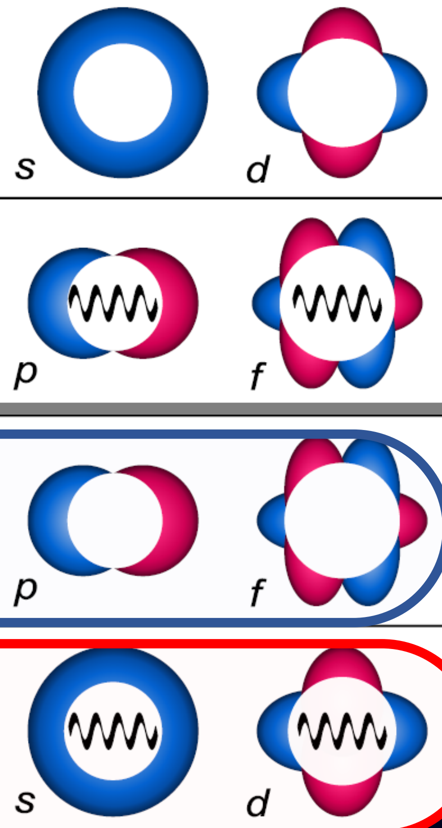


Allowed pairing symmetries

$$\Psi\ (\sigma_{1,2}\ ;\ k_{1,2}\ ;\ \omega_{1,2})$$



Spin	Frequency	Momentum
Singlet (odd) $\uparrow\downarrow - \downarrow\uparrow$	Even	Even
	Odd	Odd
Triplet (even) $\uparrow\downarrow + \downarrow\uparrow$, $\uparrow\uparrow$, $\downarrow\downarrow$	Even	Odd
	Odd	Even



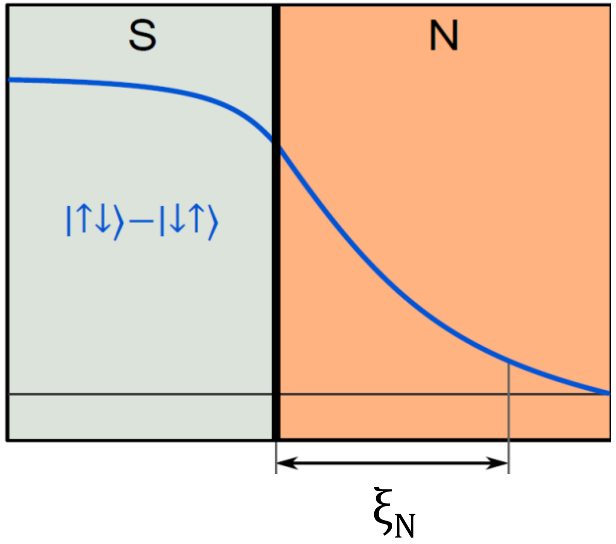
S-wave: Nb, Al, MoGe,…
d-wave: Cuprates (e.g. YBCO)

^{3}He, UPt_3

So far, only observed in S-F hybrids (via proximity effect)

Proximity Effect

$$\xi_N = \sqrt{\frac{\hbar D_N}{2\pi K_B T}} \sim \boldsymbol{\mu m}$$



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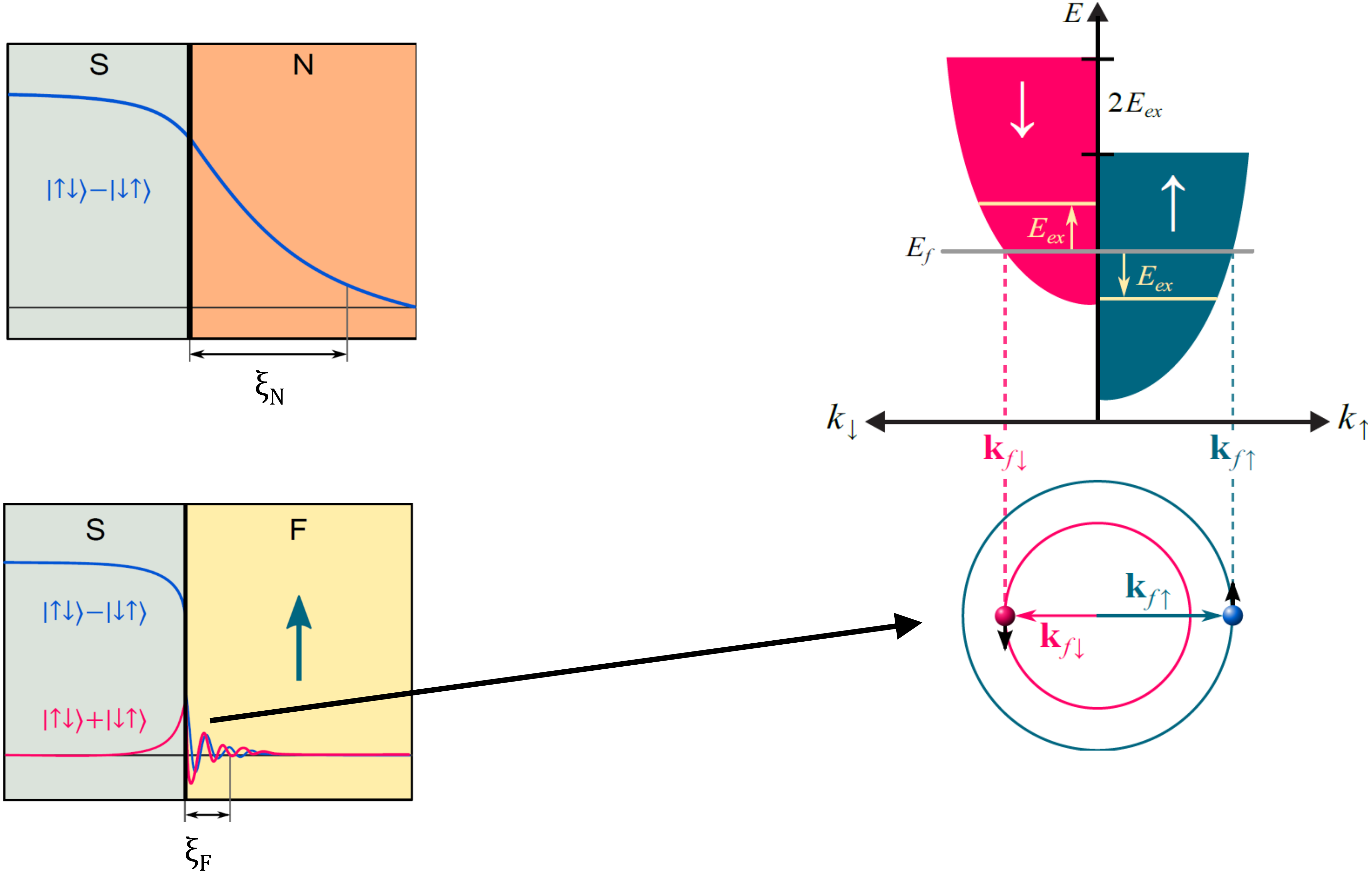

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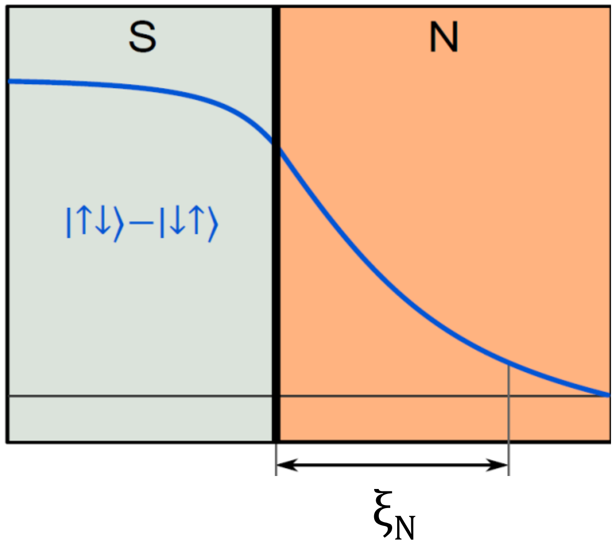
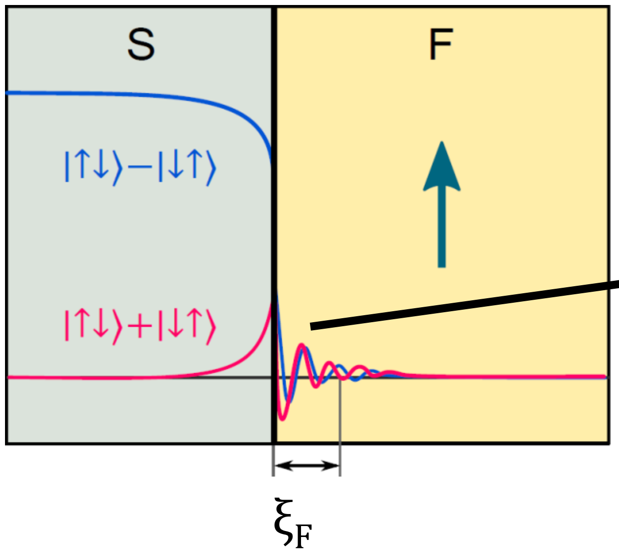
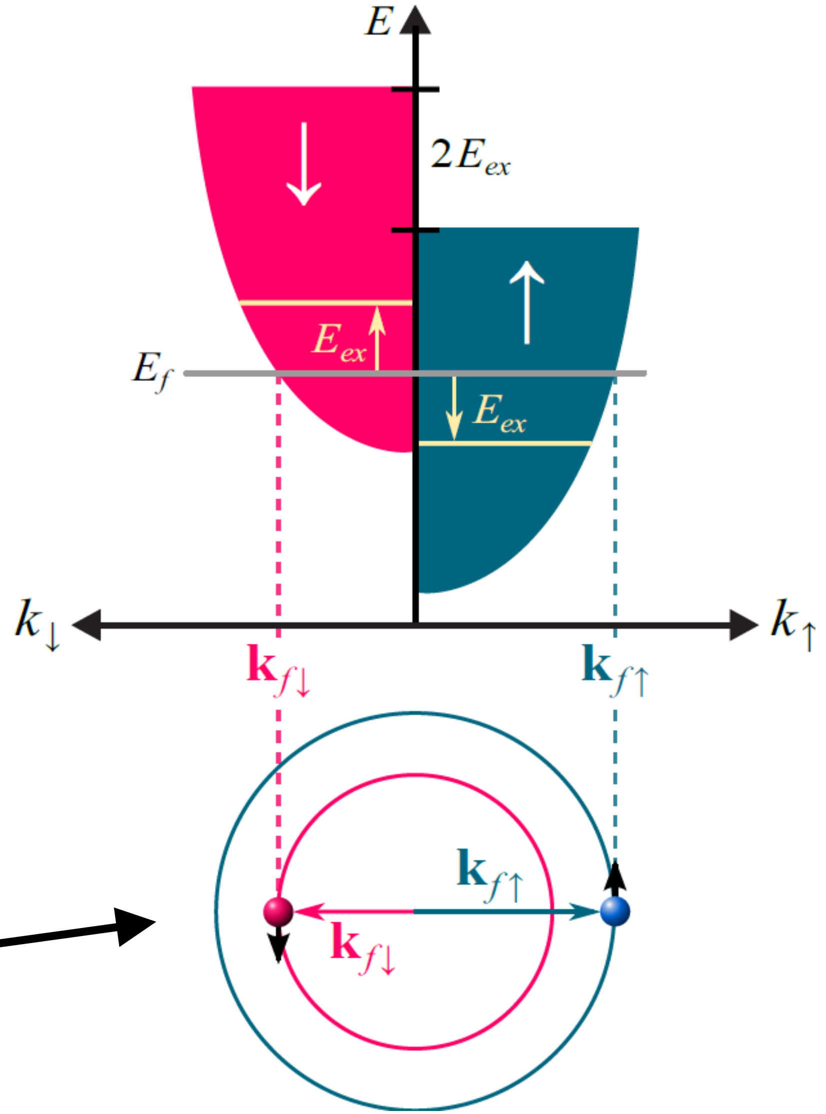


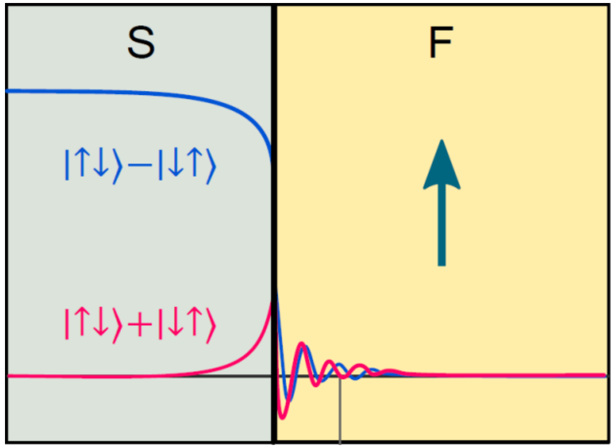
$$\xi_F = \sqrt{\frac{\hbar D_F}{2E_{ex}}} \sim \mathbf{1nm}$$

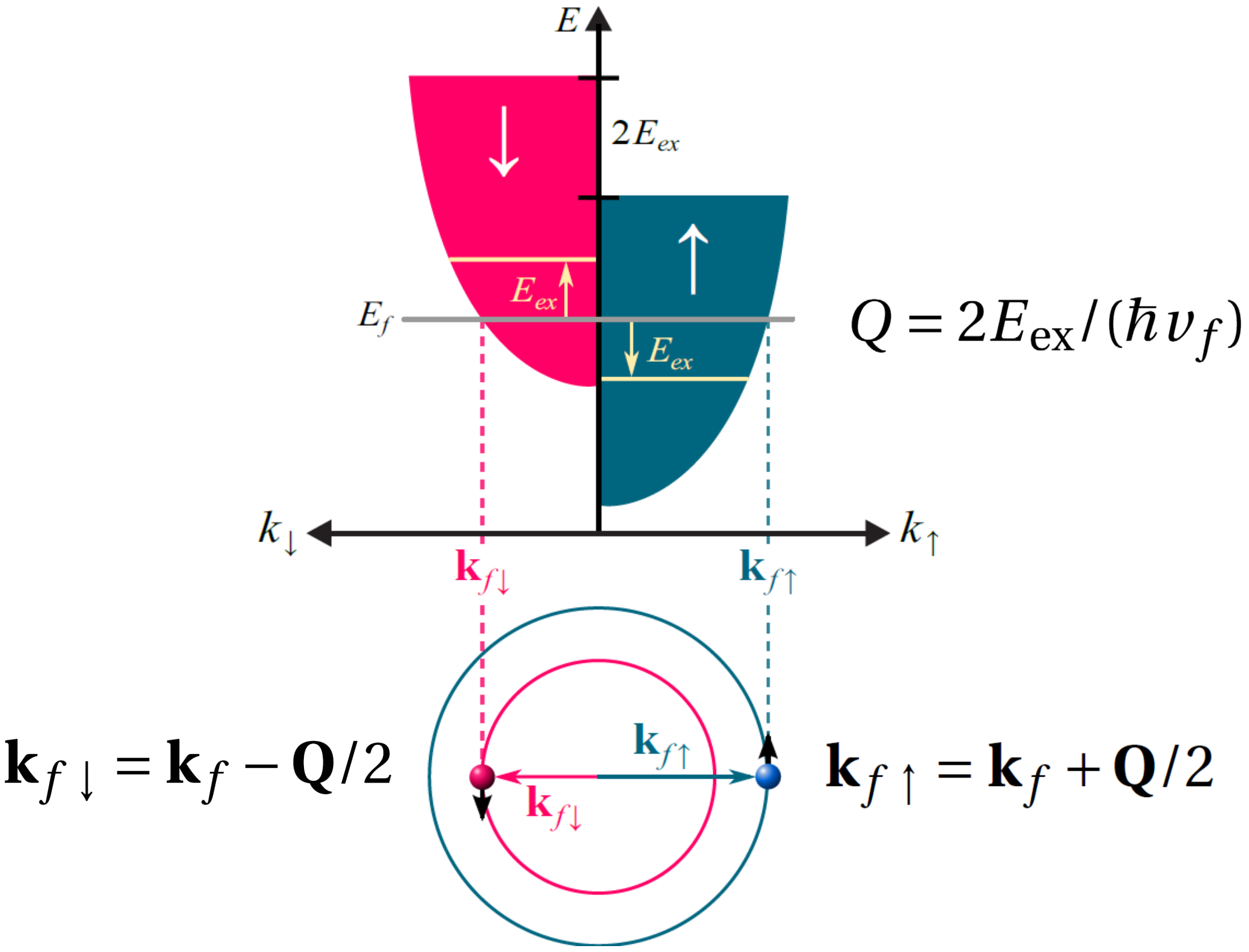




Decoherence due to the exchange field of ferromagnet: *Short range* proximity

S-F Proximity Effect: FFLO state (Cooper pairs with finite momentum)





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* Also known as the LOFF state, it is named after Peter Fulde and Richard Ferrell [3], and Anatoly Larkin and Yurii Ovchinnikov [4], who independently proposed the idea in connection with the coexistence problem of superconductivity with ferromagnetism.

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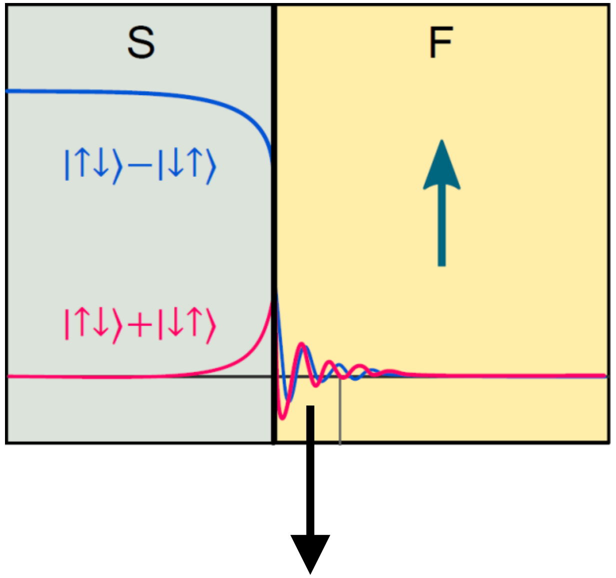
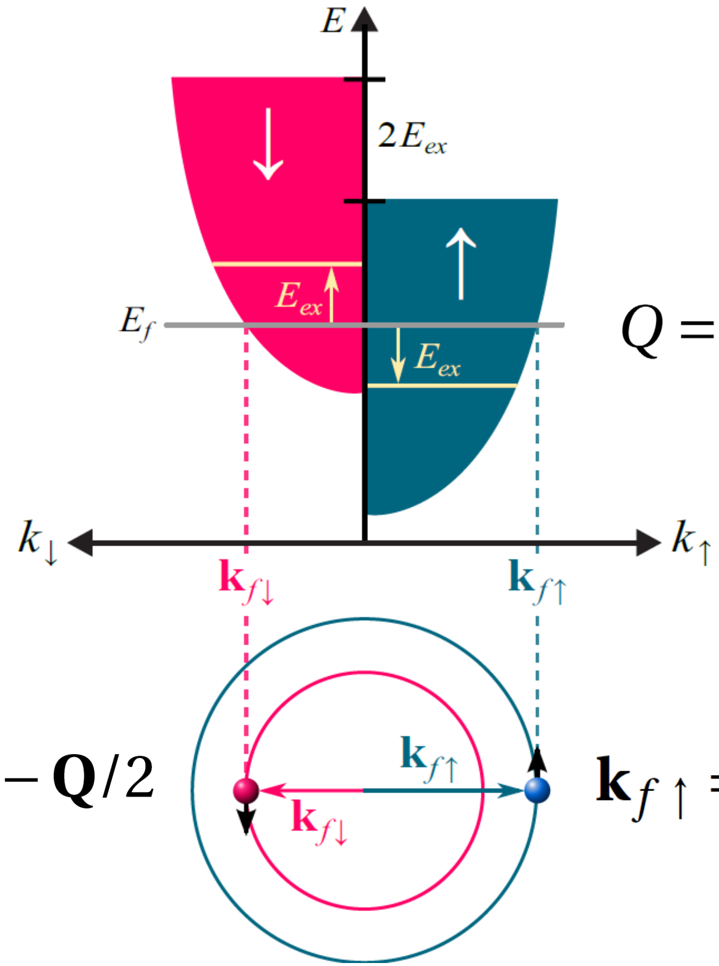


$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \longrightarrow \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle e^{i\mathbf{R}.\mathbf{Q}} - |\downarrow\uparrow\rangle e^{-i\mathbf{R}.\mathbf{Q}}\right)$$



$$Q = 2E_{\mathrm{ex}}/(\hbar v_f)$$

$$\mathbf{k}_{f\downarrow} = \mathbf{k}_f - \mathbf{Q}/2 \qquad \mathbf{k}_{f\uparrow} = \mathbf{k}_f + \mathbf{Q}/2$$

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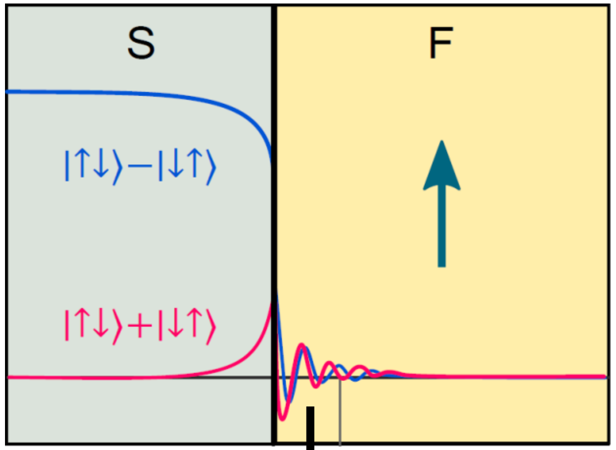

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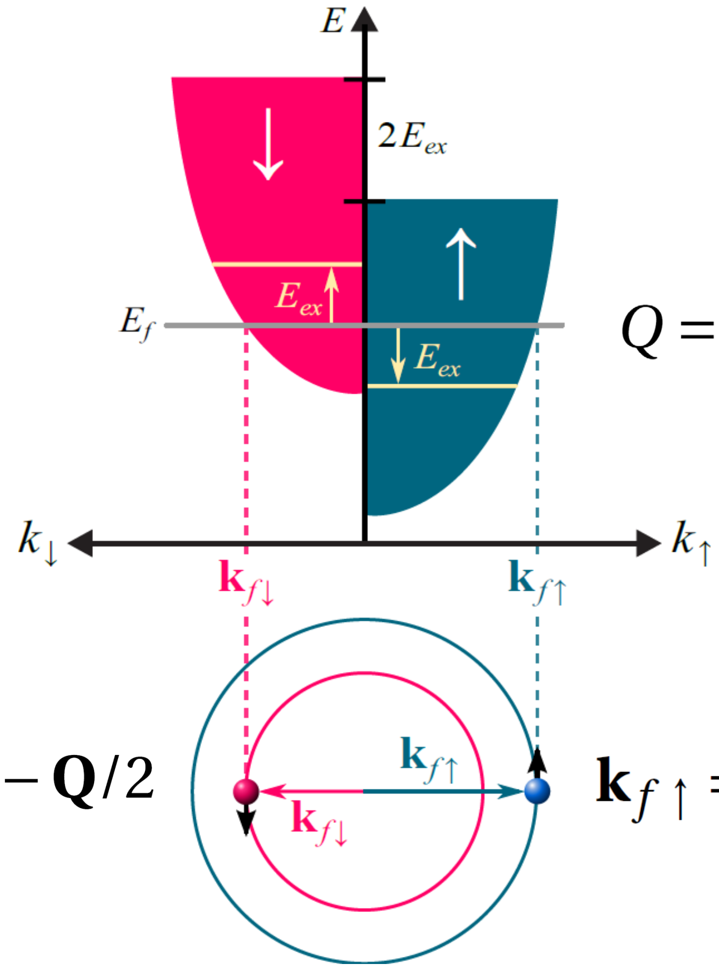
$$\frac{1}{\sqrt{2}}\left[\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)\cos(\mathbf{R}.\mathbf{Q}) \quad + \quad i\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\sin(\mathbf{R}.\mathbf{Q})\right]$$



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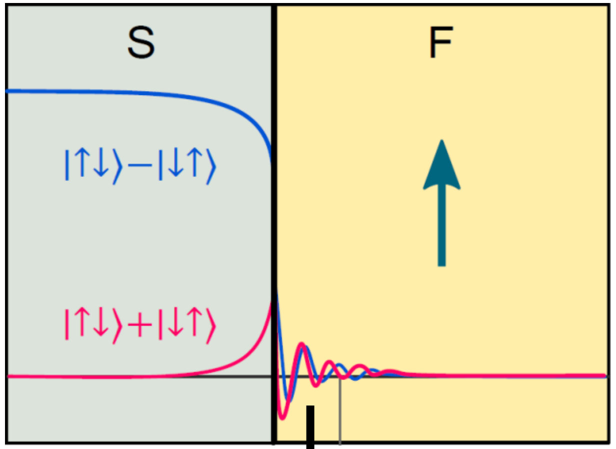

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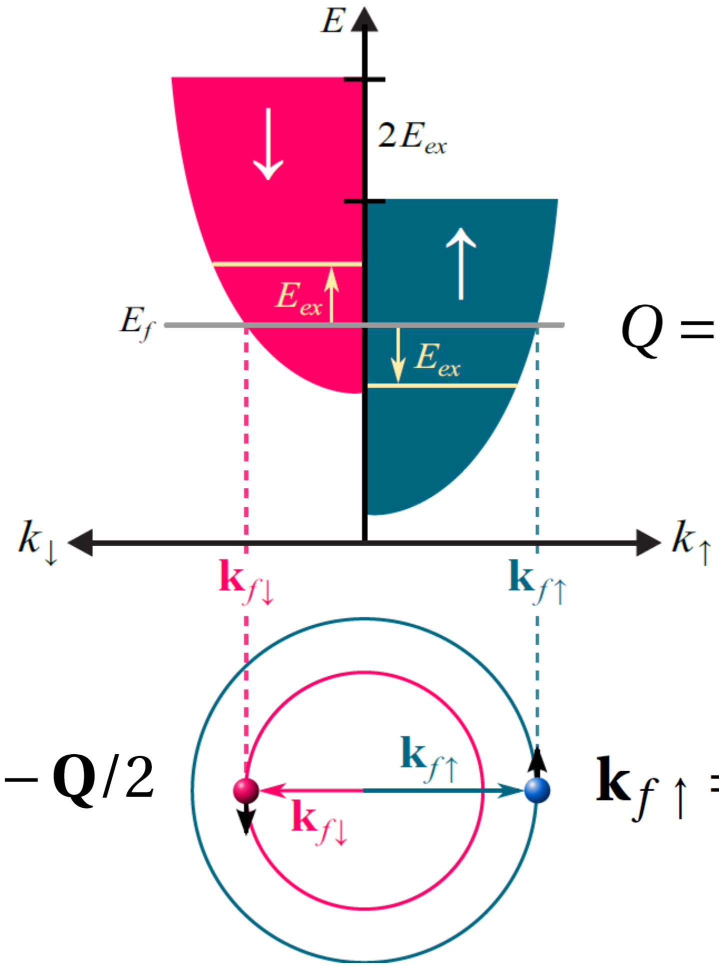
Singlet

Triplet (m=0)



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$$\mathbf{k}_{f\downarrow} = \mathbf{k}_f - \mathbf{Q}/2 \qquad \mathbf{k}_{f\uparrow} = \mathbf{k}_f + \mathbf{Q}/2$$

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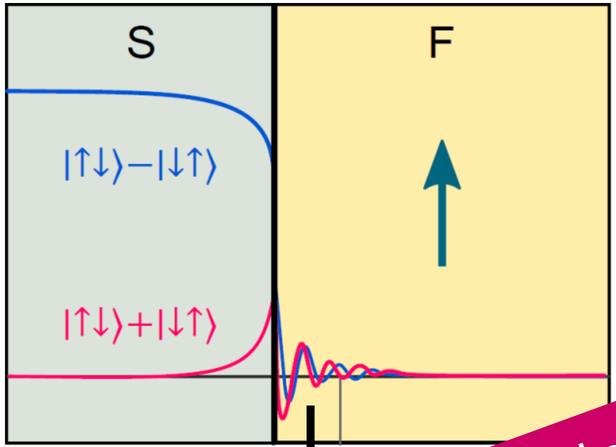
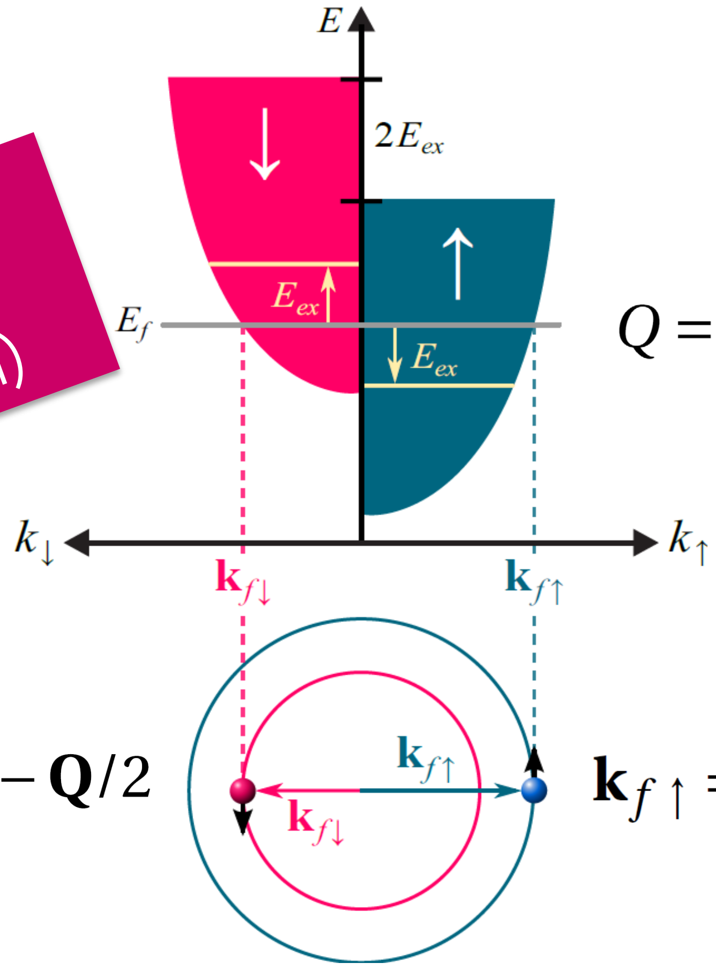
S-F Proximity Effect: FFLO state (Cooper pairs with finite momentum)





$$Q = 2E_{\text{ex}}/(\hbar v_f)$$

$$\mathbf{k}_{f\downarrow} = \mathbf{k}_f - \mathbf{Q}/2 \qquad \mathbf{k}_{f\uparrow} = \mathbf{k}_f + \mathbf{Q}/2$$

Just like singlets, m=0 triplets are suppressed by the exchange field of the ferromagnet, Resulting in short-range proximity effect (~1-10nm)

$$\frac{1}{\sqrt{2}}\left[\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)\cos(\mathbf{R}.\mathbf{Q}) \quad + \quad i\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\sin(\mathbf{R}.\mathbf{Q})\right]$$

Singlet

Triplet (m=0)

* Also known as the LOFF state, it is named after Peter Fulde and Richard Ferrell [3], and Anatoly Larkin and Yurii Ovchinnikov [4], who independently proposed the idea in connection with the coexistence problem of superconductivity with ferromagnetism.

Long-range SF proximity: Spin-polarized Cooper pairs

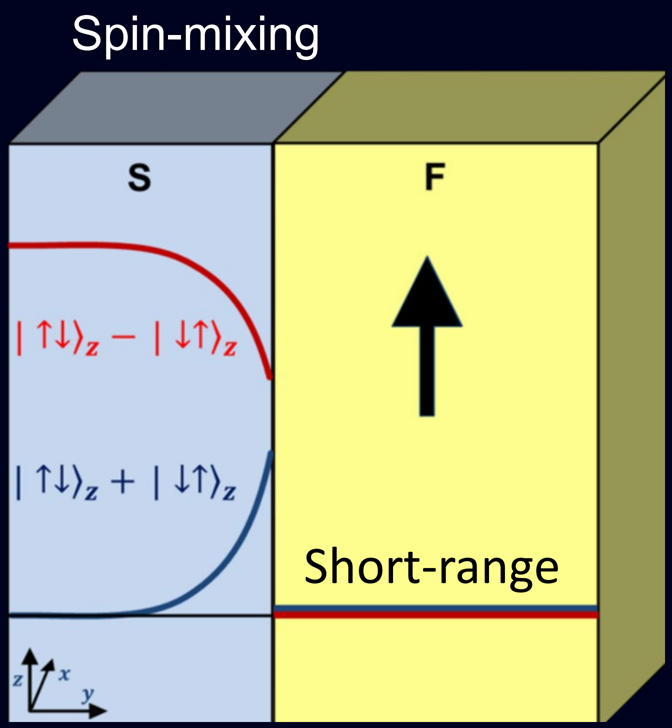
$|\uparrow\uparrow\rangle$ $\quad m_z = +1$

$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ $\quad m_z = 0$ $\quad z$

$|\downarrow\downarrow\rangle$ $\quad m_z = -1$

$$\xi_F = \sqrt{\frac{\hbar D_F}{2\mathrm{Eex}}}$$

ξ_F (Co) ~ 5 nm

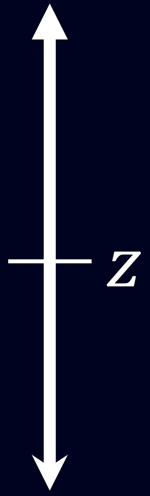


Long-range SF proximity: Spin-polarized Cooper pairs

$| \uparrow\uparrow \rangle$ $\quad m_z = +1$

$| \uparrow\downarrow \rangle + | \downarrow\uparrow \rangle$ $\quad m_z = 0$

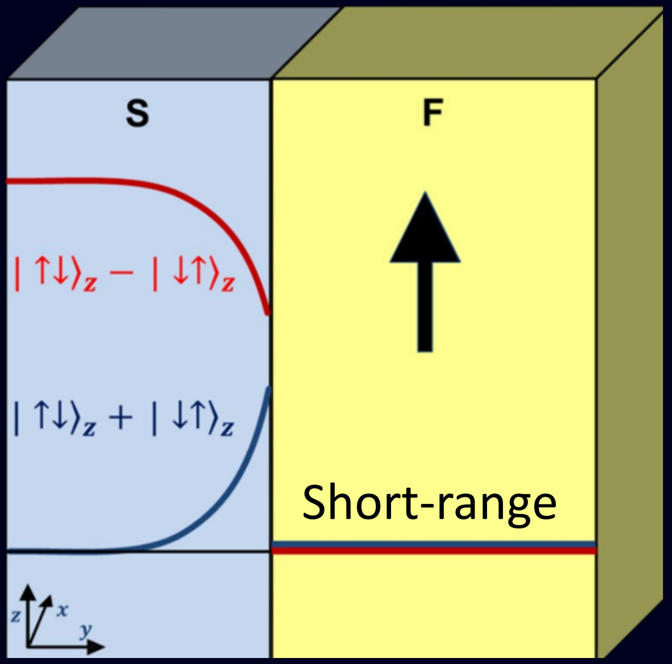
$| \downarrow\downarrow \rangle$ $\quad m_z = -1$



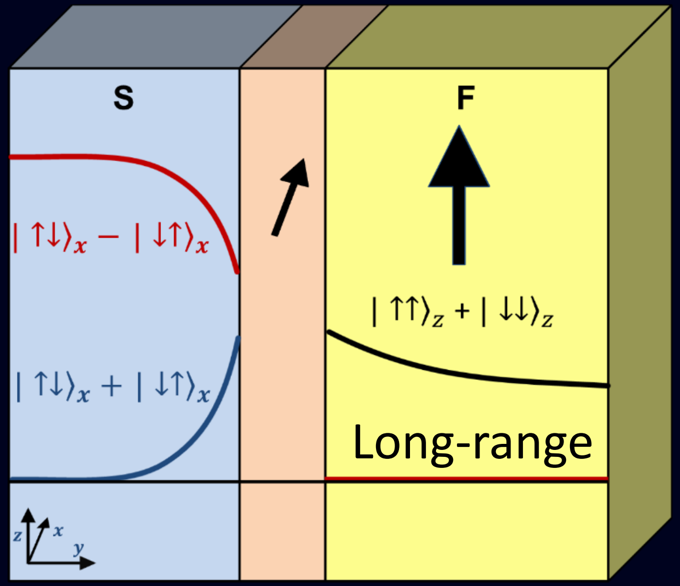
$$\xi_F = \sqrt{\frac{\hbar D_F}{2\mathrm{Eex}}}$$

ξ_F (Co) ~ 5 nm

Spin-mixing



Spin-mixing + *rotation*



Long-range SF proximity: Spin-polarized Cooper pairs

$|\uparrow\uparrow\rangle$ $\quad m_z = +1$

$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ $\quad m_z = 0$

$|\downarrow\downarrow\rangle$ $\quad m_z = -1$

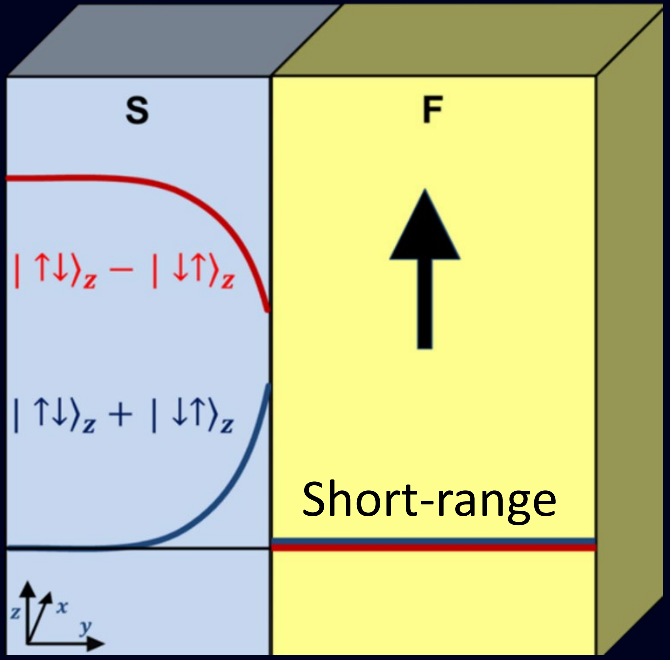
z

$$\xi_F = \sqrt{\frac{\hbar D_F}{2Eex}}$$

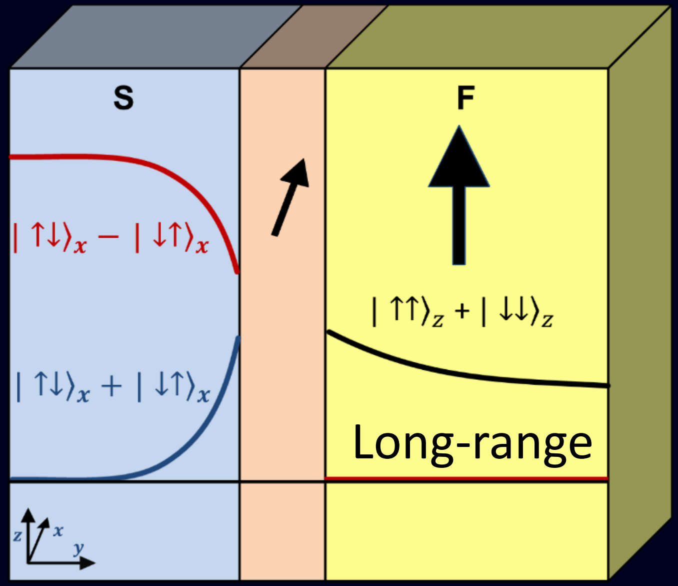
ξ_F (Co) $\sim$ 5 nm

$\xi_F^T \sim$ 100s nm

Spin-mixing



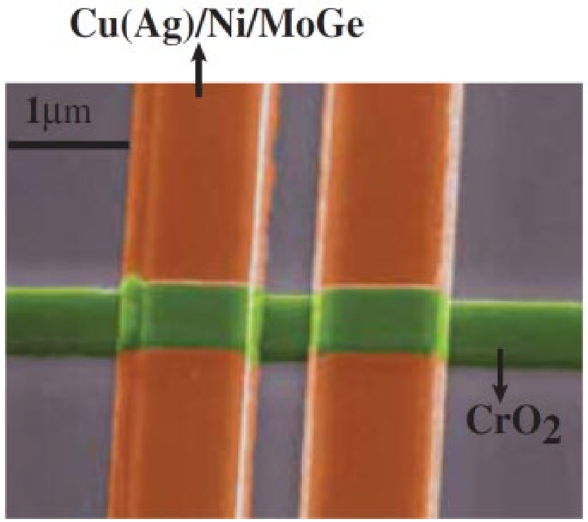
Spin-mixing + *rotation*



~100% spin polarized

700 nm CrO_2 wire

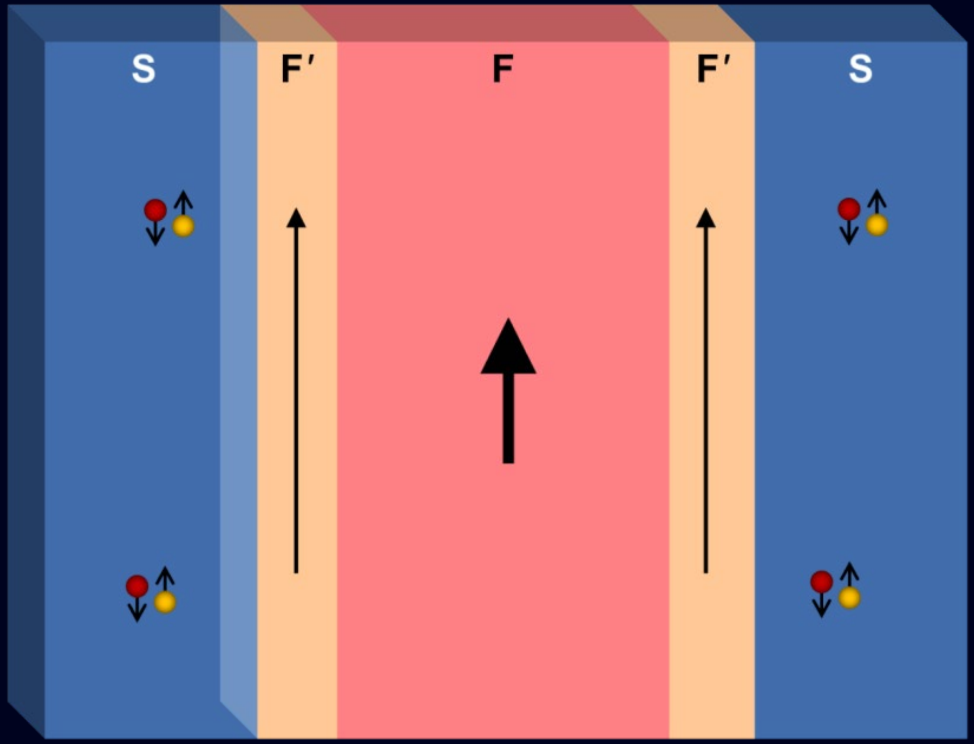
$J_c \sim$ **10^9 A/m^2!**



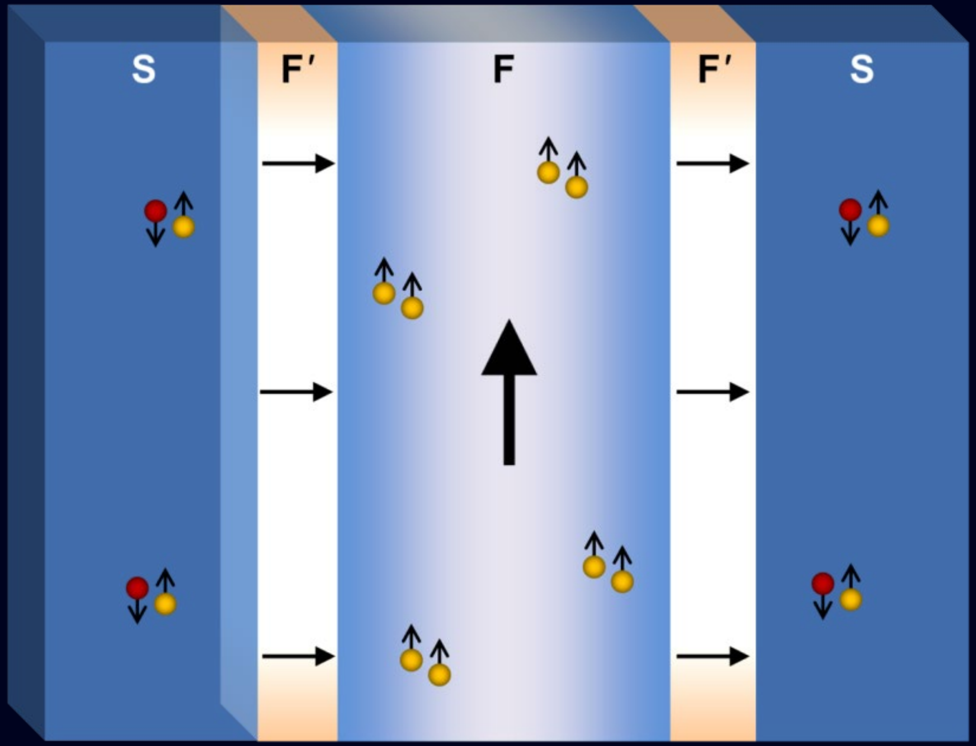
Singh, Aarts *et al.*, PRX, (2016)

Break?

Superconducting (dissipation-less) spintronics:



supercurrent **off**



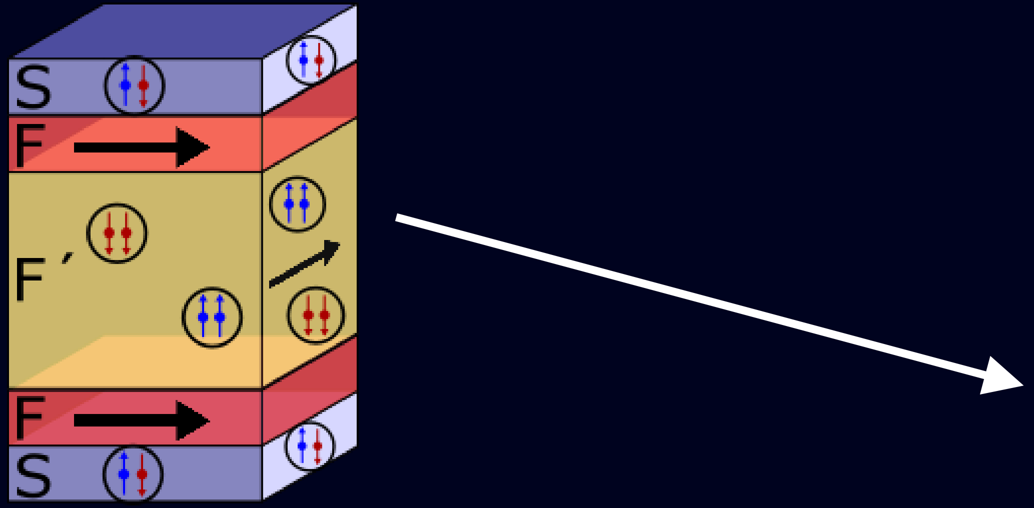
supercurrent **on**

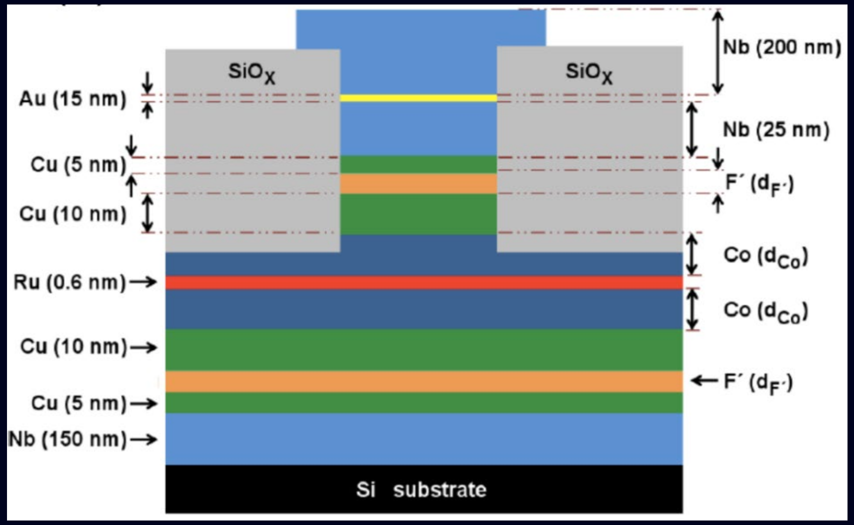
Josephson junctions with non-collinear F layers

Device configuration (concept)

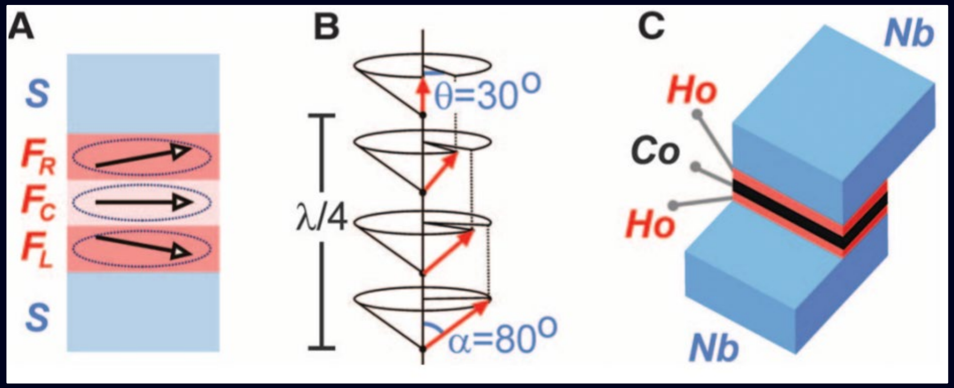
Actual devices

Stack





Khaire *et al.* PRL **104** (2010)

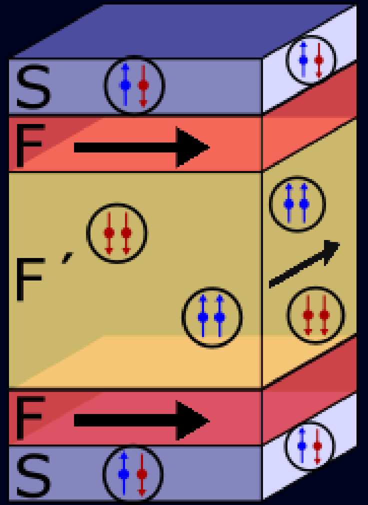


Robinson *et al.* Science, **329** (2010)

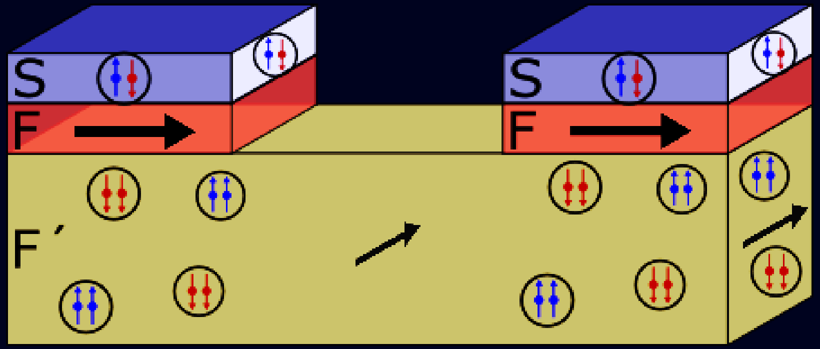
Josephson junctions with non-collinear F layers

Device configuration (concept)

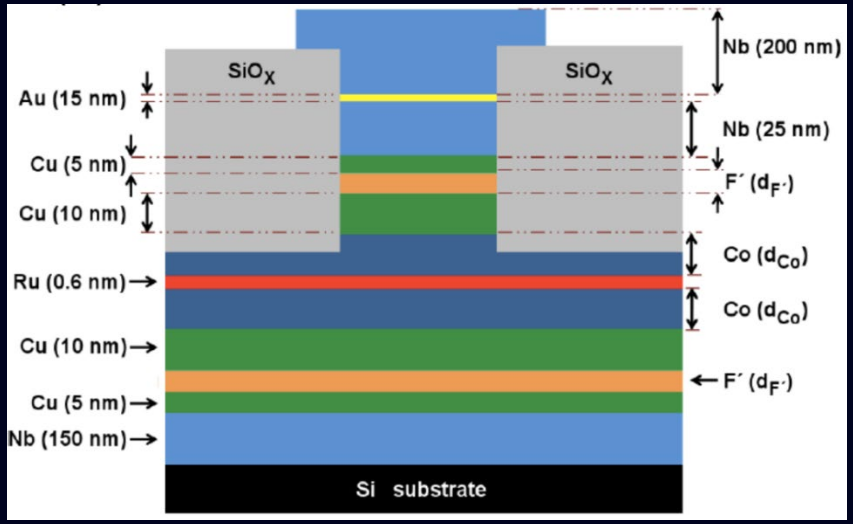
Stack



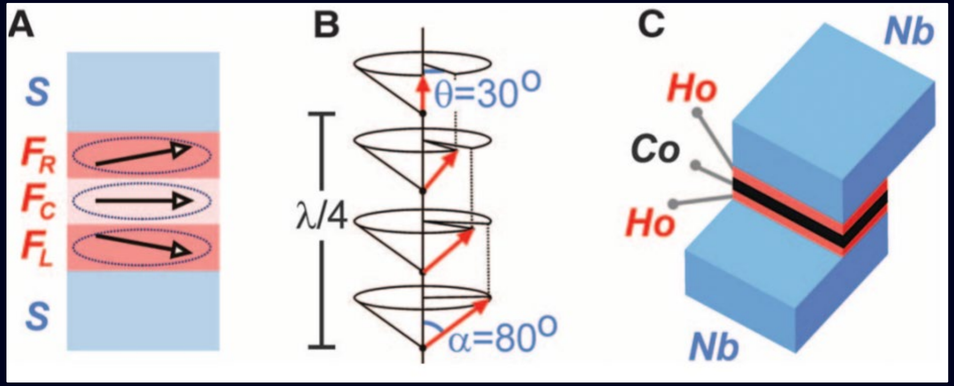
Planar?



Actual devices



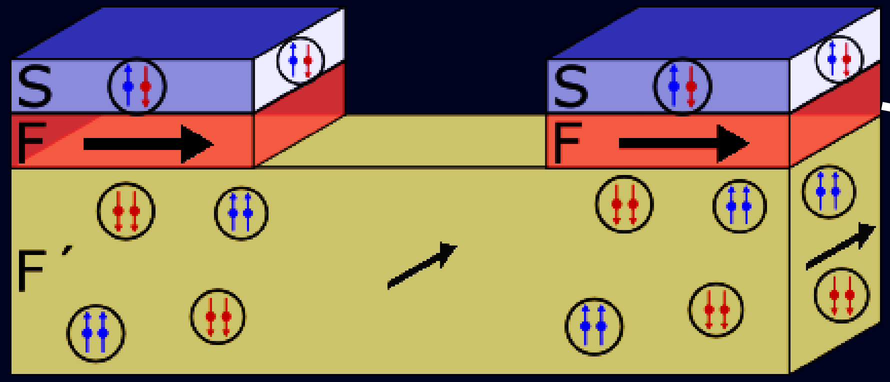
Khaire *et al.* PRL **104** (2010)

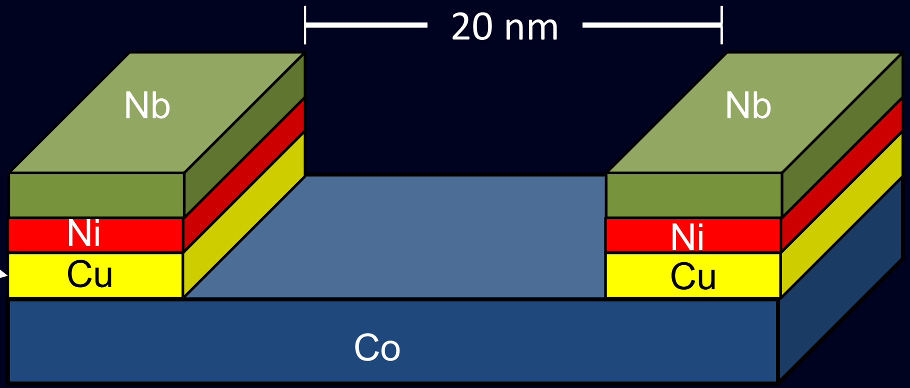


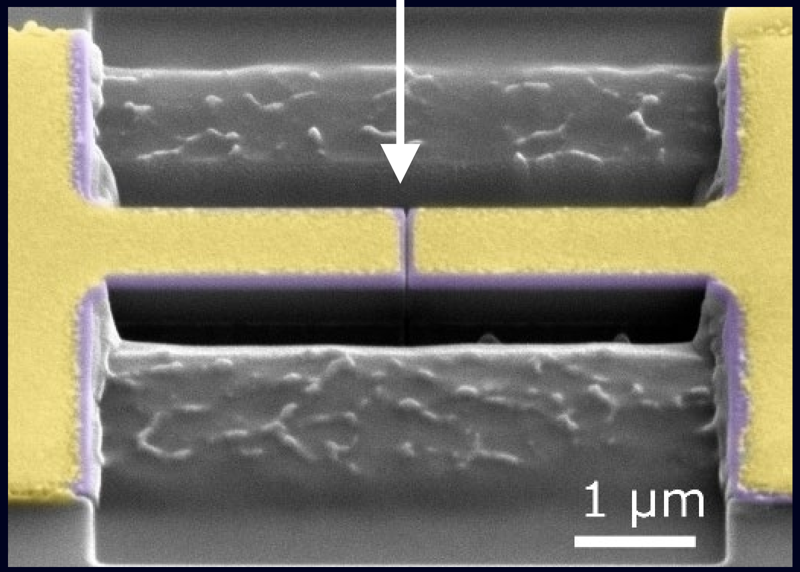
Robinson *et al.* Science, **329** (2010)

Decoupling the magnetic layers

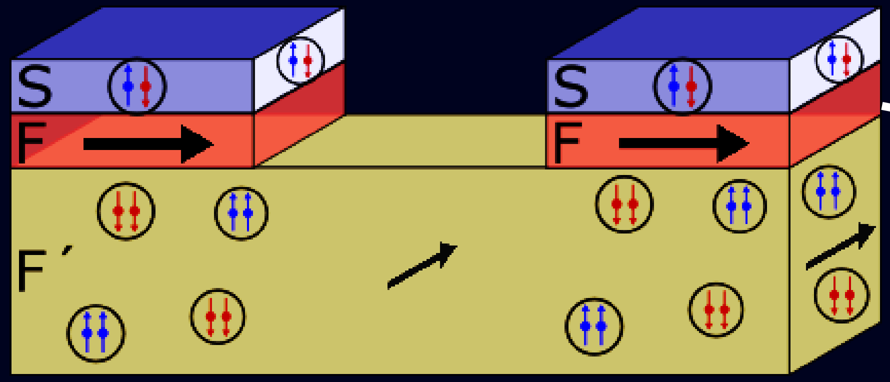
Decoupling the magnetic layers

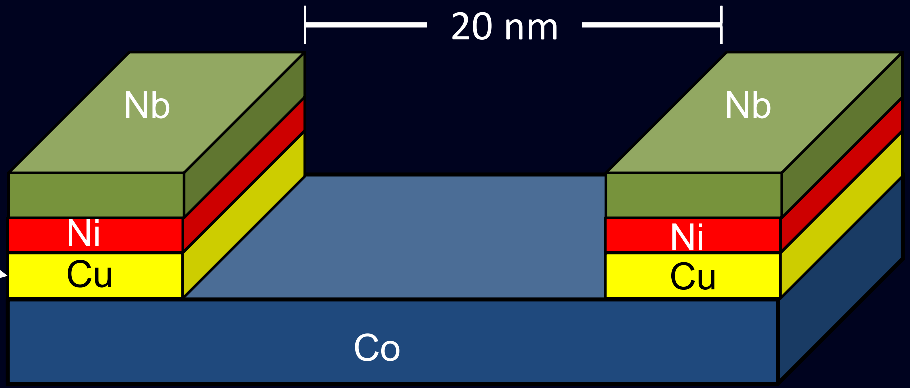


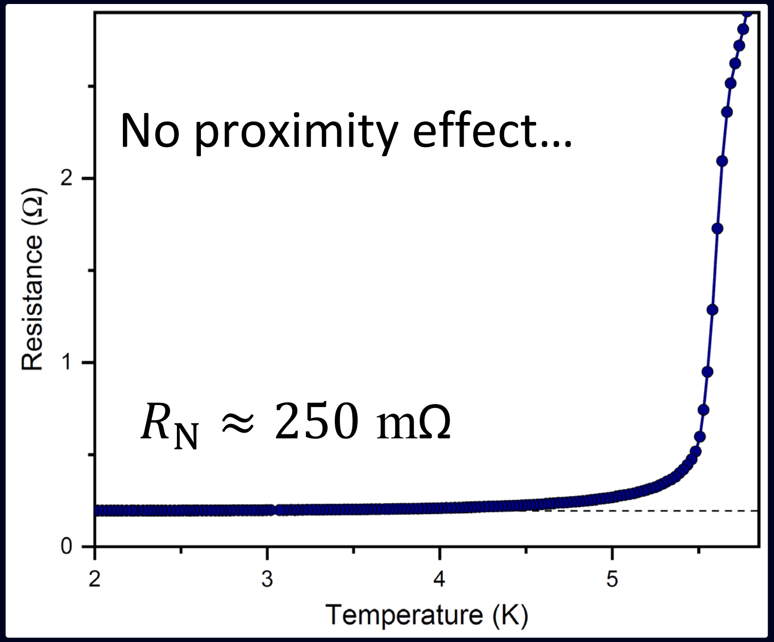


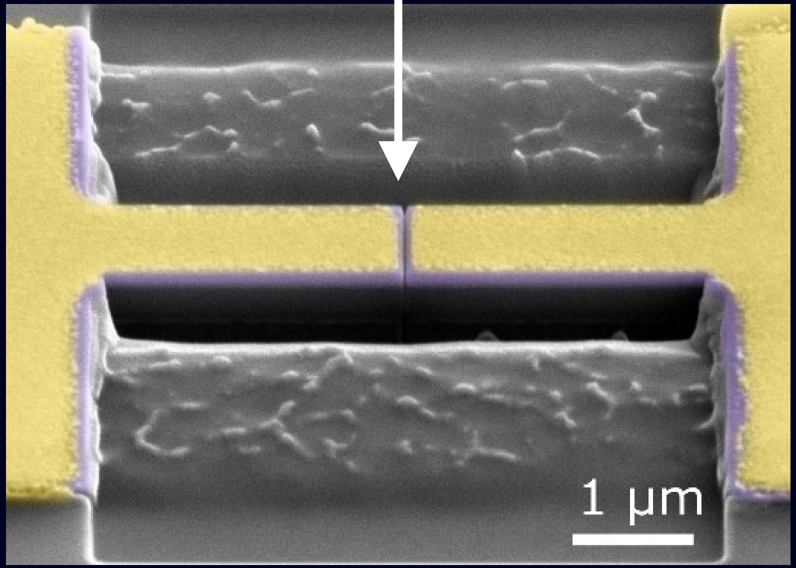


Decoupling the magnetic layers

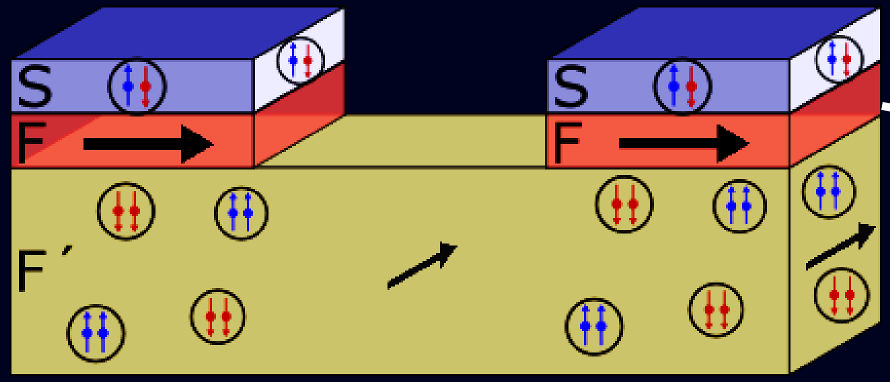


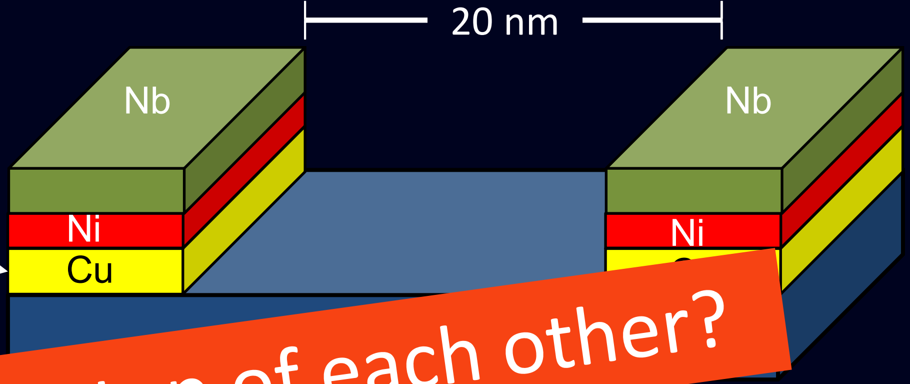




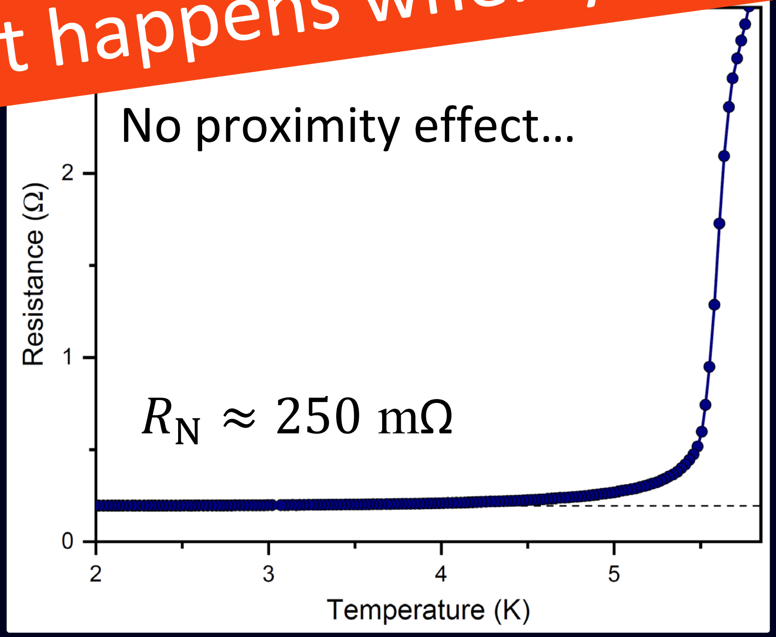


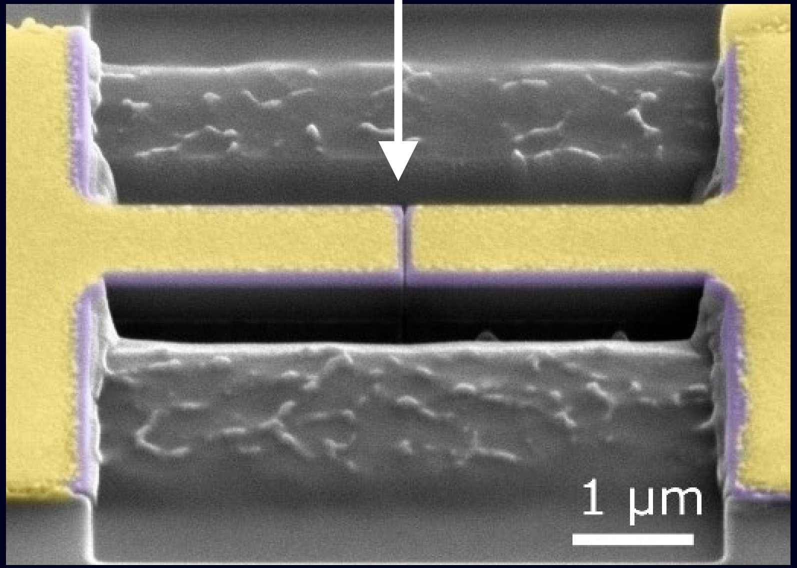
Decoupling the magnetic layers



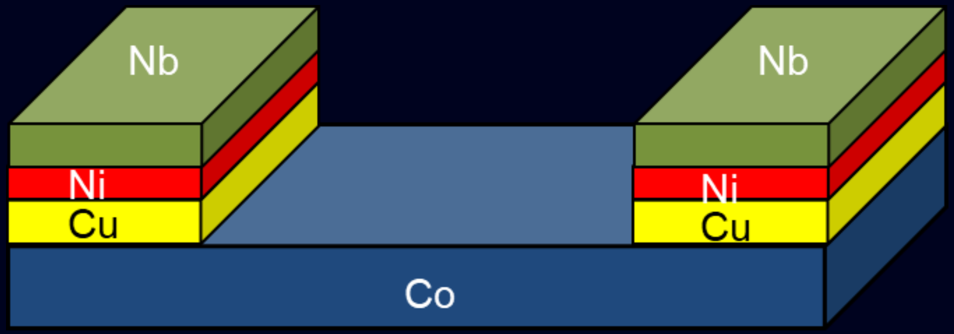


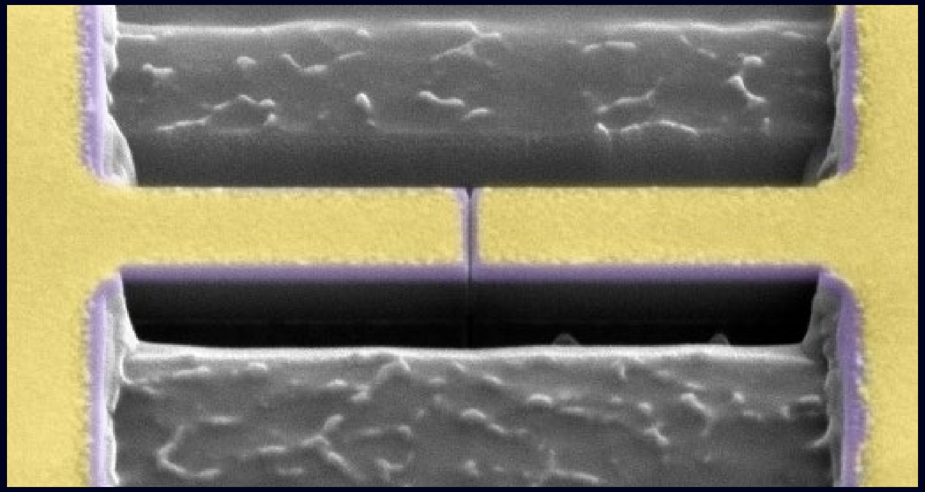
What happens when you put two F layers on top of each other?



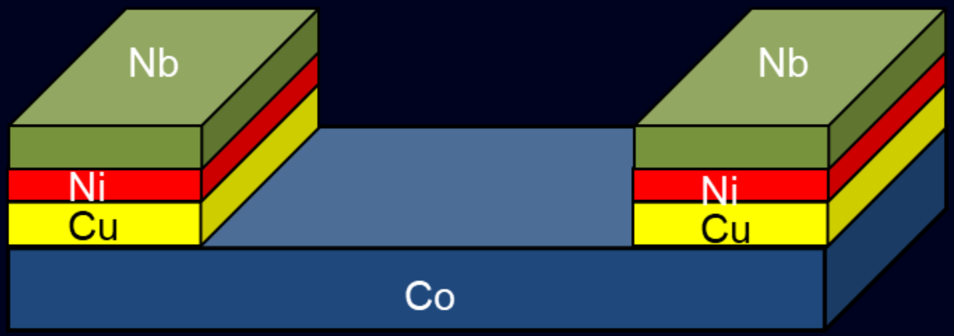


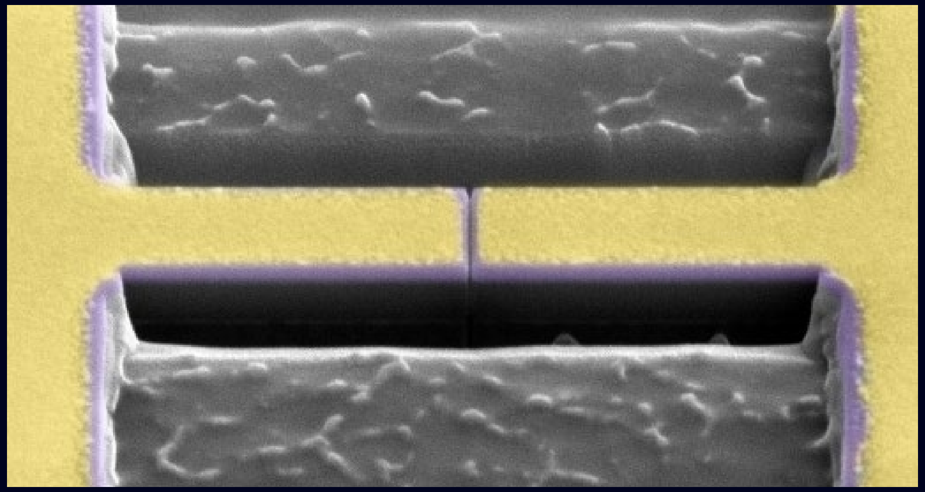
Let’s simulate the magnetic layers

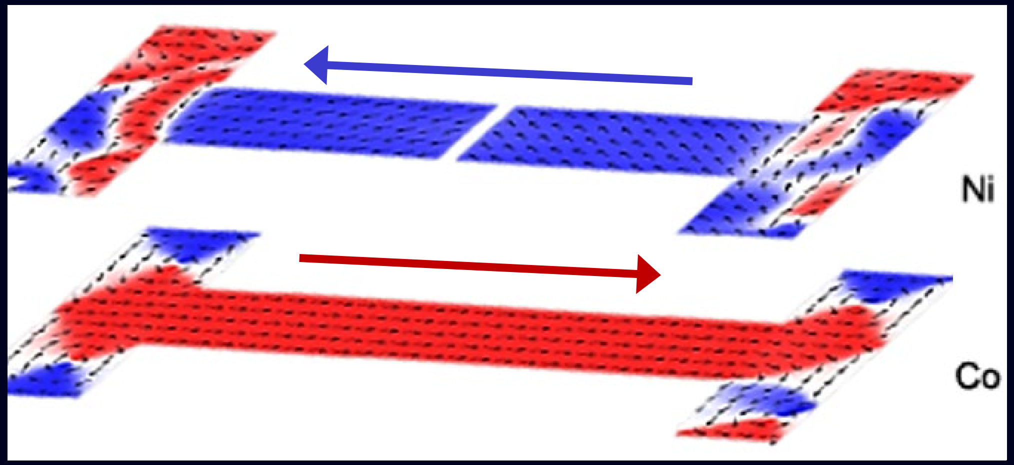




Let's simulate the magnetic layers

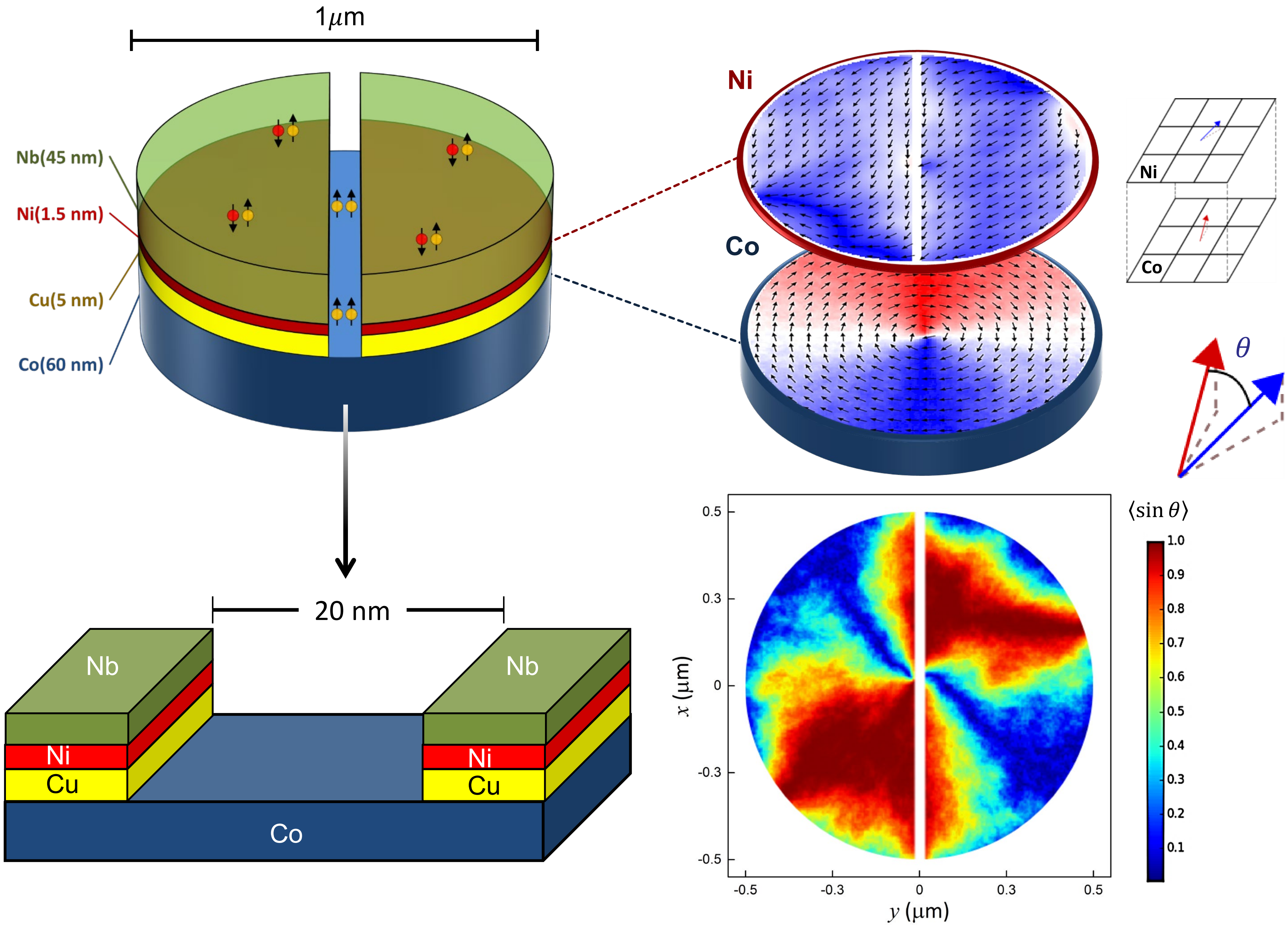






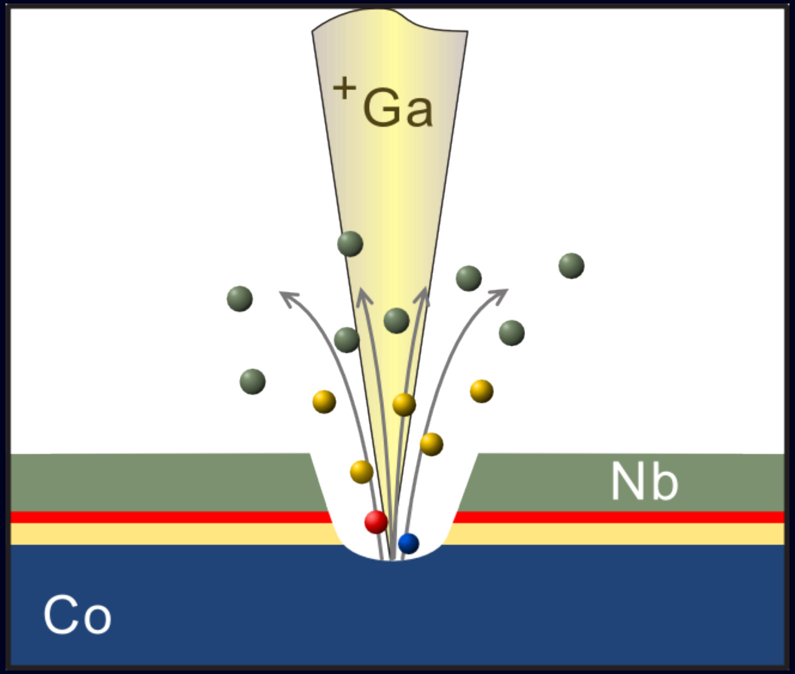
F layers have antiparallel magnetization.

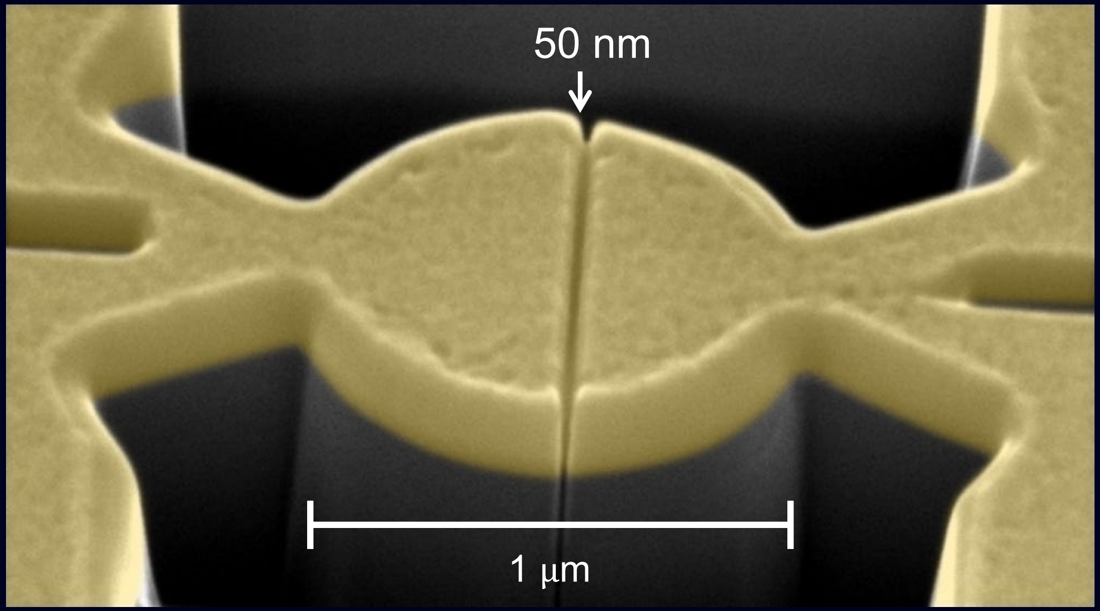
Why is this energetically favorable?



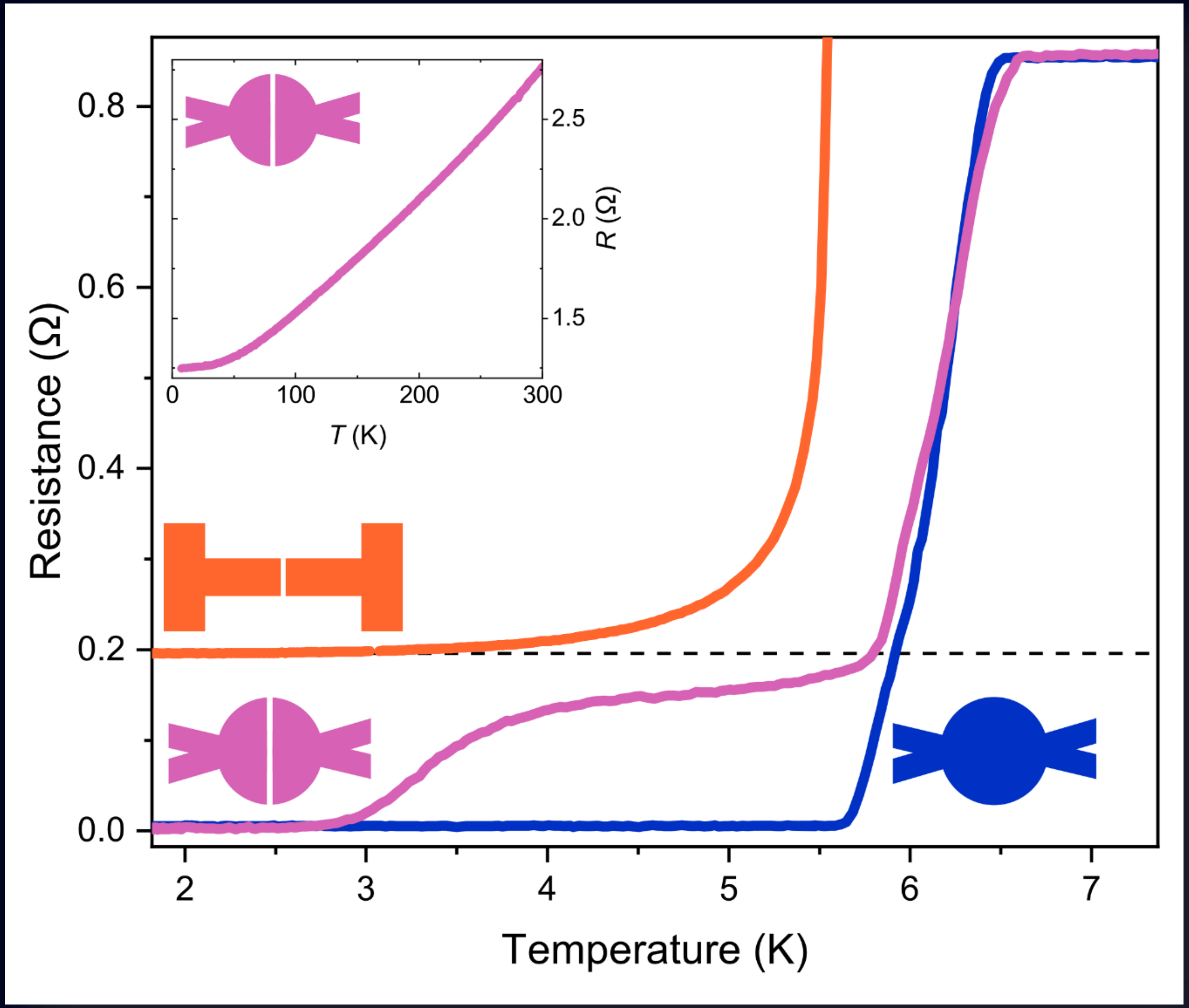
Lahabi *et al.*, Nat. Comm **8** (2017)

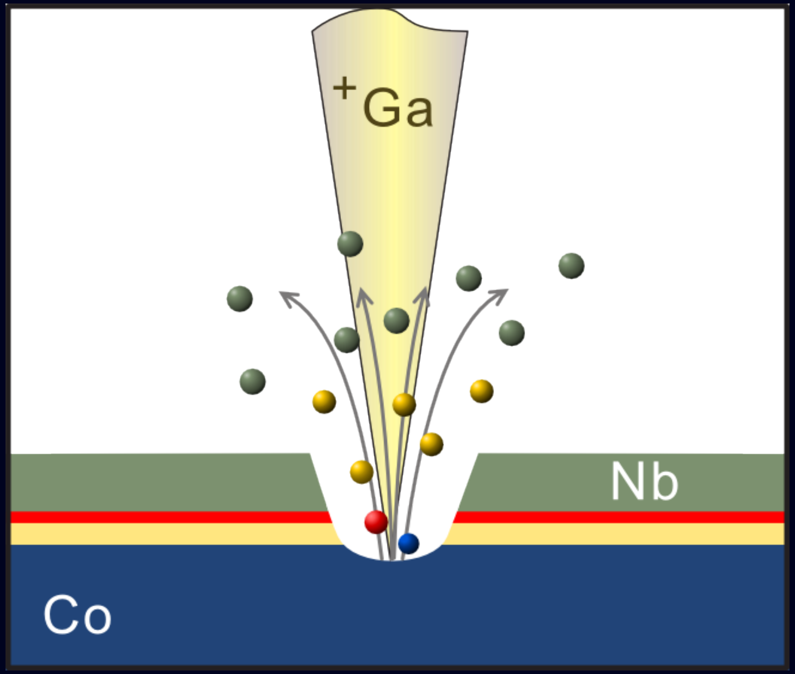
Nanostructured by $^{+}$Ga focused ion beam

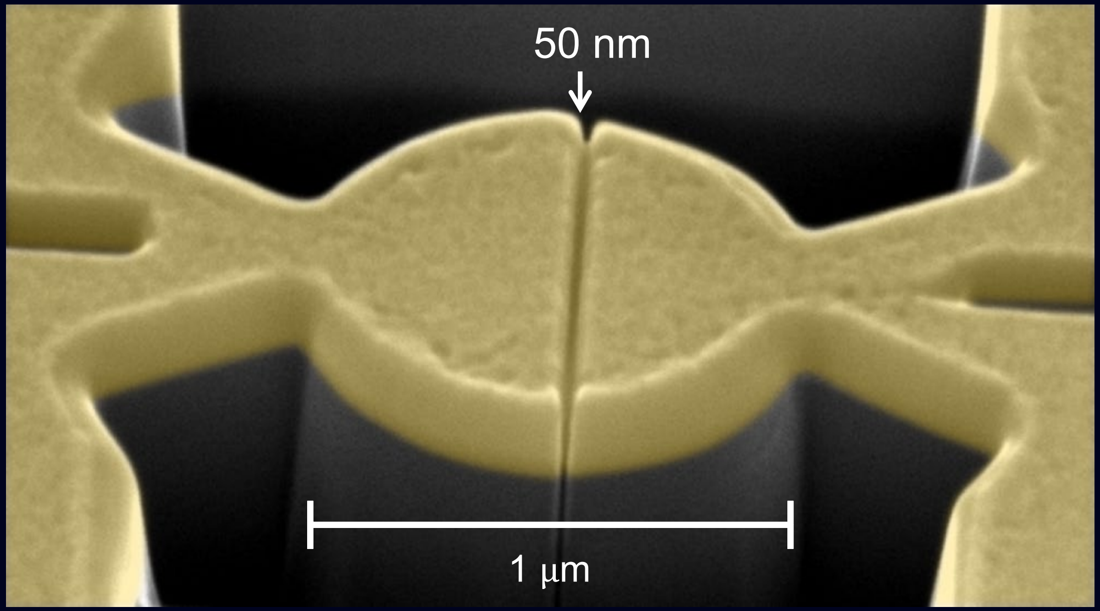




Nanostructured by $^{+}$Ga focused ion beam







More on Josephson junctions next time

End of Lecture 4

REVIEWS OF MODERN PHYSICS, VOLUME 75, APRIL 2003

The superconductivity of Sr_2RuO_4 and the physics of spin-triplet pairing

Andrew Peter Mackenzie

School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SS, Scotland

Yoshiteru Maeno

Department of Physics, Kyoto University, Kyoto 606-8502, Japan and International Innovation Center, Kyoto University, Kyoto 606-8501, Japan

A lot of the material covered here can be found in

1. 2003 review by Mackenzie & Maeno

2. Kaveh Lahabi's PhD thesis: Chapter 2 &3 (scan the QR)

Kaveh Lahabi (2025)

Chapter 2

Chapter 3