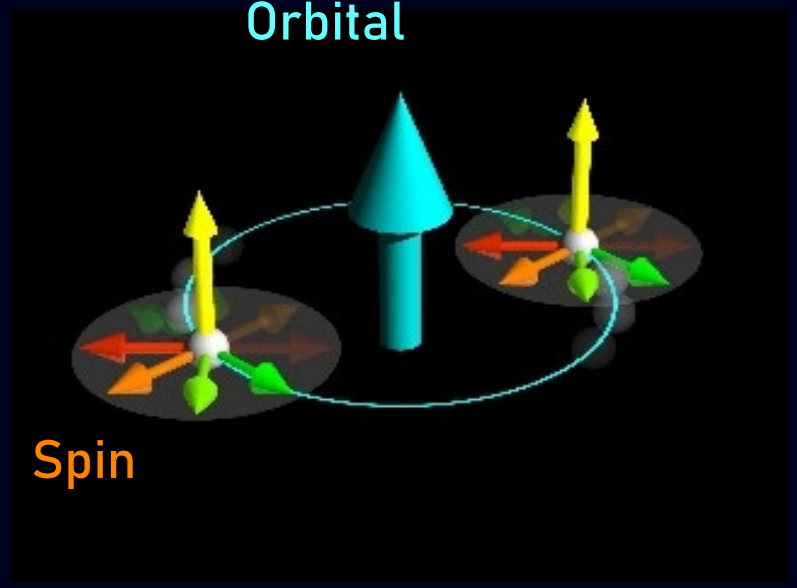
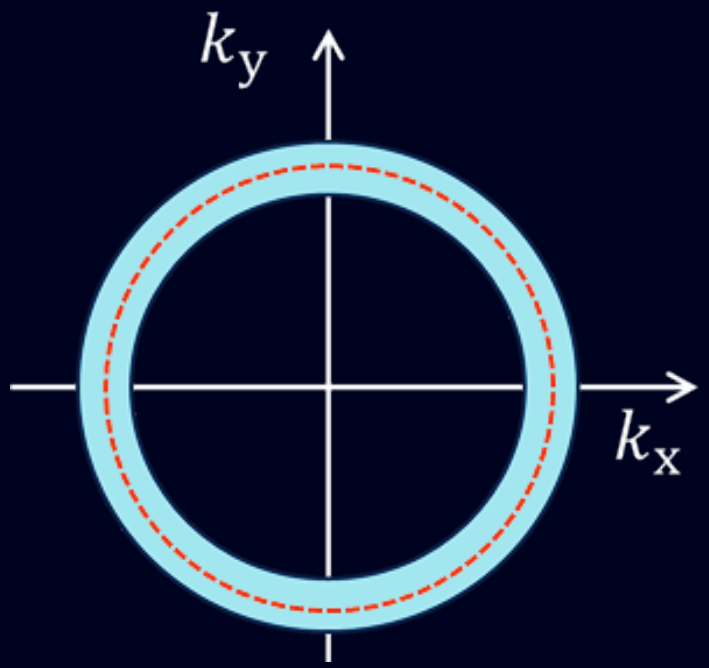
Superconductivity Lecture 3: Symmetries

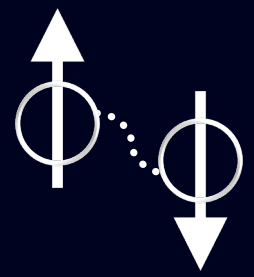




Kaveh Lahabi (2025)

Cooper pairs spin symmetry

$$\mathbf{S} = \mathbf{0}$$



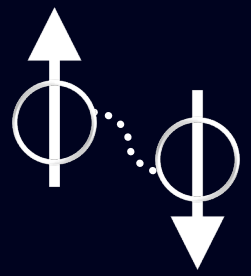
Singlet pairing (opposite spins)

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Cooper pairs spin symmetry

S = spin
m = spin *projection* wrt the quantization axis

$\mathbf{S = 0}$



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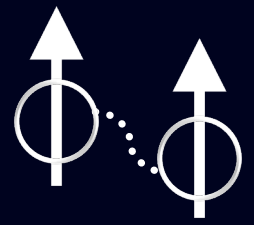

? $\mathbf{S = 1}$

Equal spin Cooper pairs?

Triplet pairing

$$|\uparrow\uparrow\rangle \qquad m_z = +1$$

$$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \qquad m_z = 0$$

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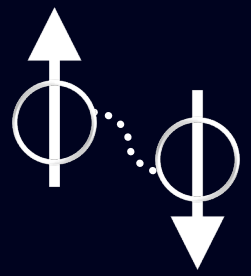

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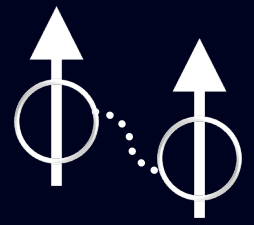 ? $\mathbf{S = 1}$

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What about Pauli's principle?

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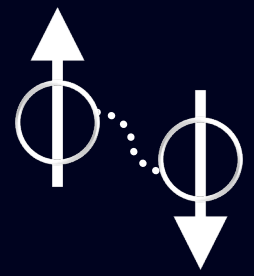
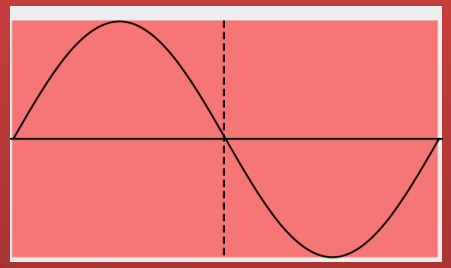


Singlet pairing (opposite spins)

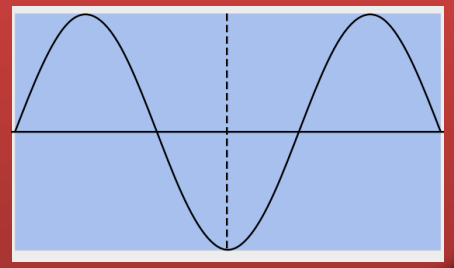
$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

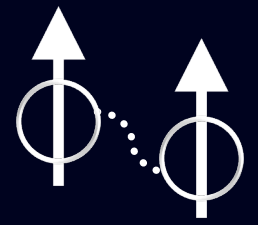
$\Psi\,(\sigma_{1,2}\,;$ ▩ $;$ ▩ $)$

odd



even





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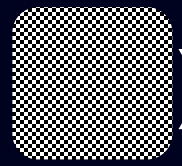
Pairing Symmetry:

$\Psi\ (\sigma_{1,2}\ ;\ k_{1,2}\ ;$  $)$

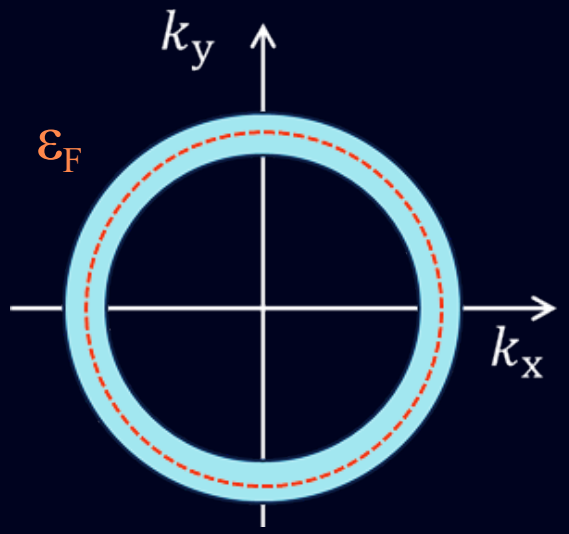
Momentum (space)

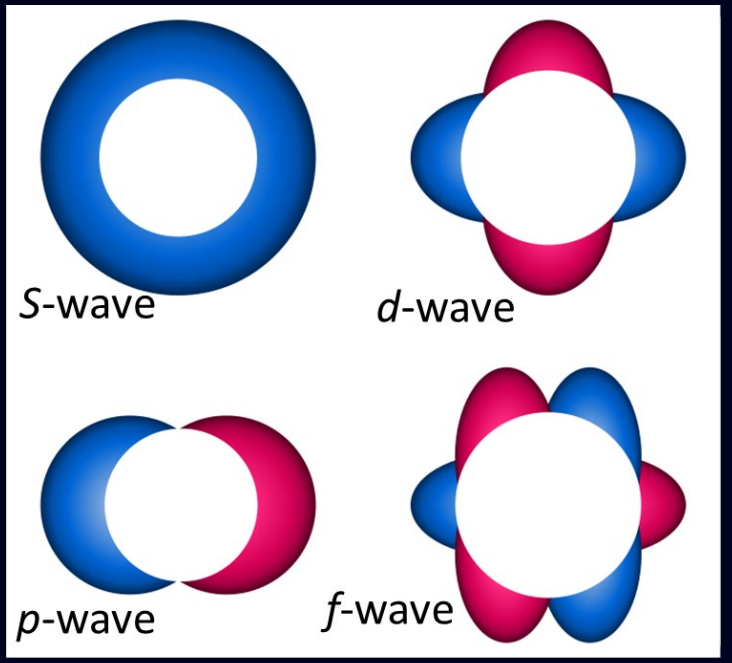
Pairing Symmetry:

$\Psi\ (\sigma_{1,2}\ ;\ k_{1,2}\ ;$ [redacted] $)$

↑

Momentum (space)





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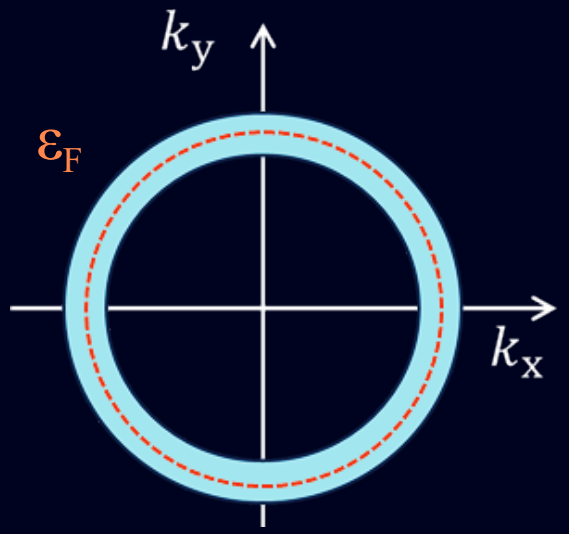
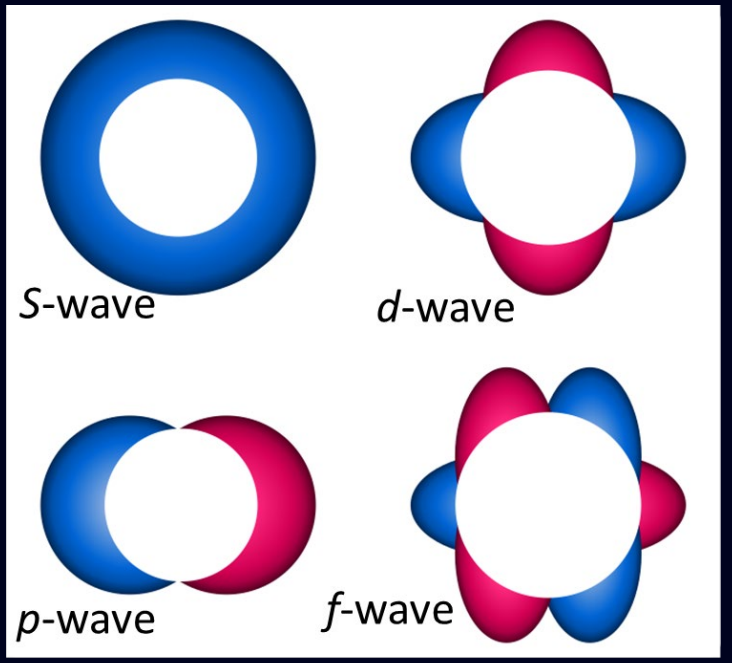


Even

Odd



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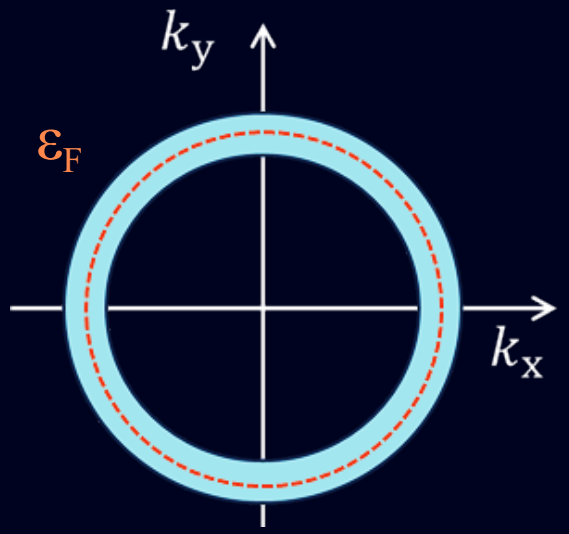

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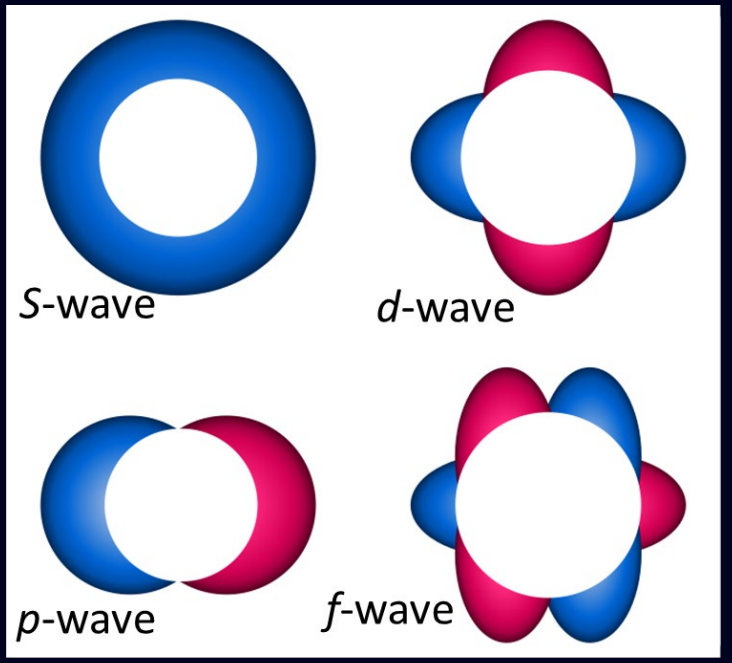

The 1st non s-wave SC discovered?

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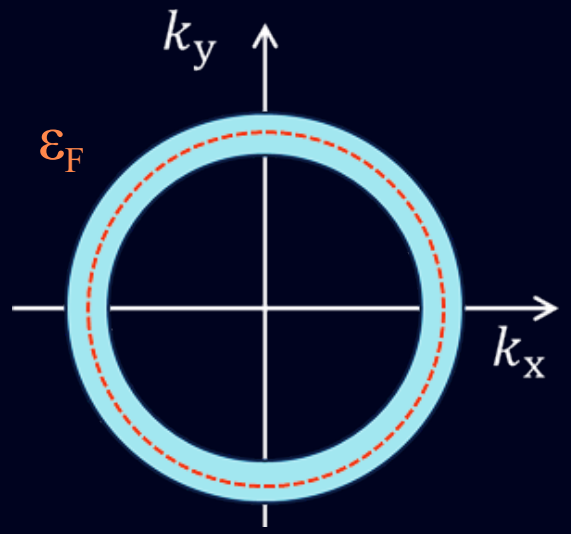

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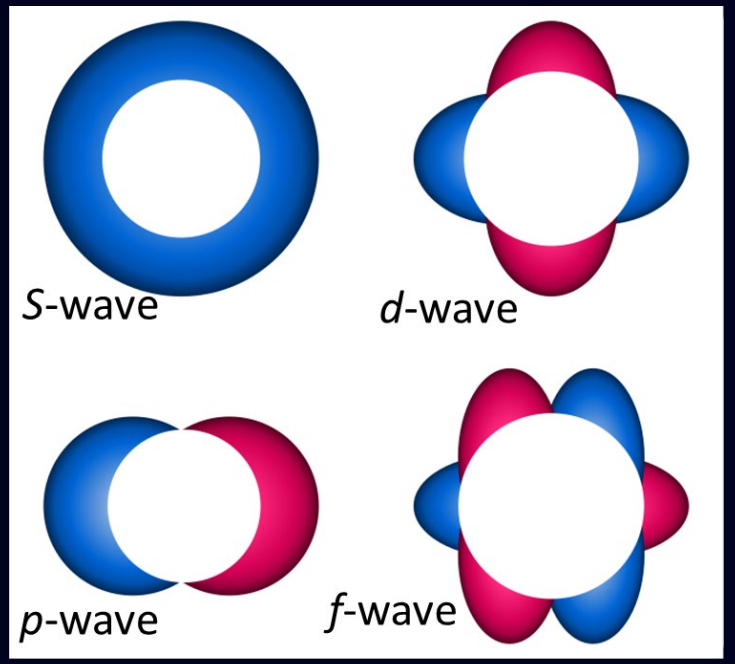

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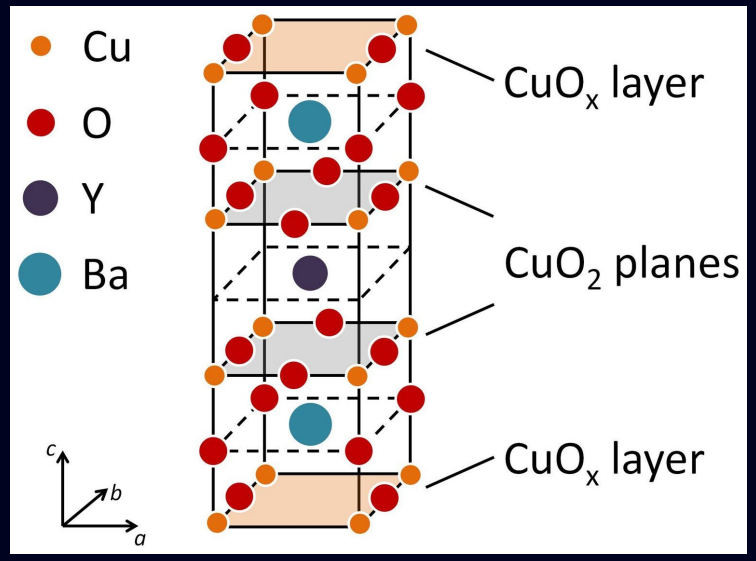
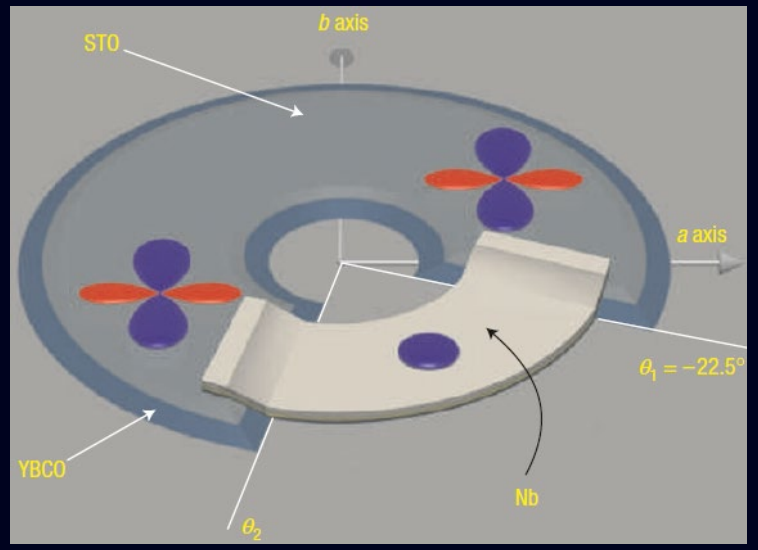
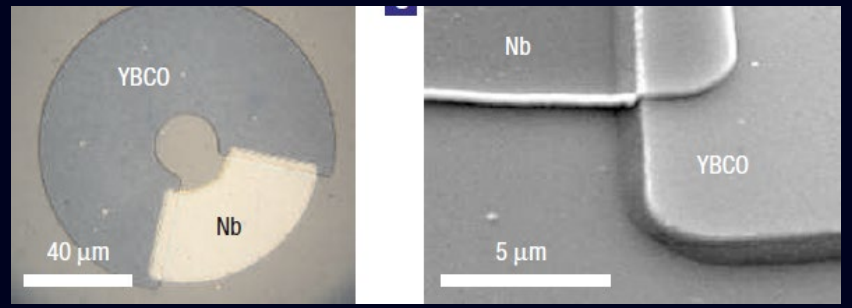


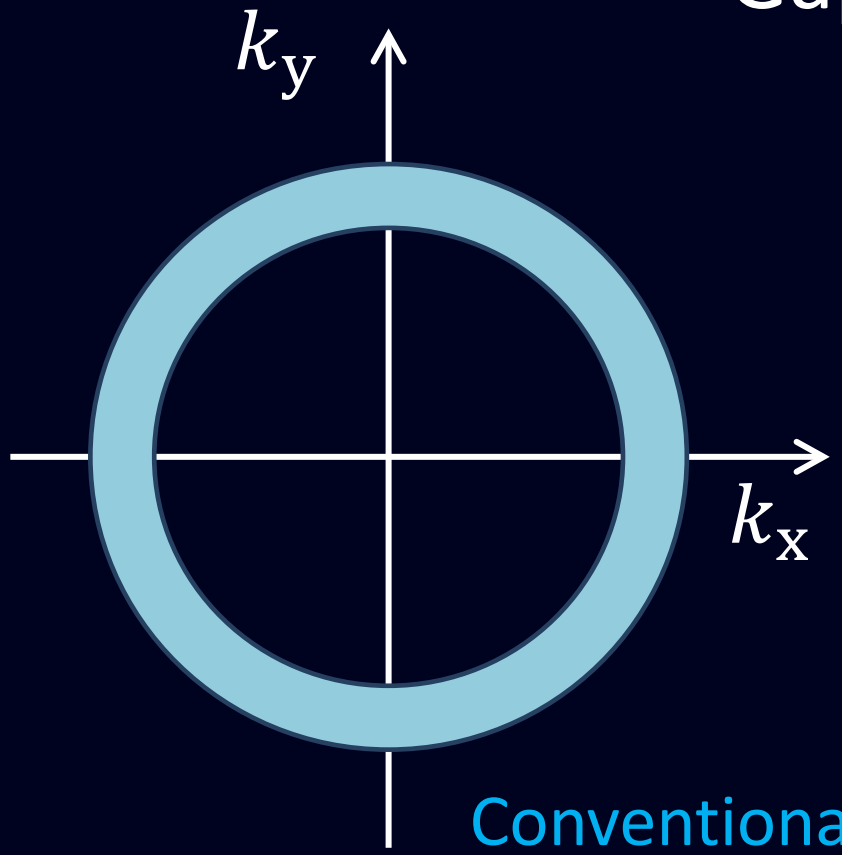




Angle-resolved phase-sensitive determination of the in-plane gap symmetry in $YBa_2Cu_3O_{7-\delta}$

J. R. KIRTLEY[1]*, C. C. TSUEI[1], A. ARIANDO[2], C. J. M. VERWIJS[2], S. HARKEMA[2] AND H. HILGENKAMP[2]

Gap parity (k-symmetry)



Conventional SC

$$\Delta(\mathbf{k}) = \Delta_0$$

Gap is k-***independent***
(some small variations allowed)

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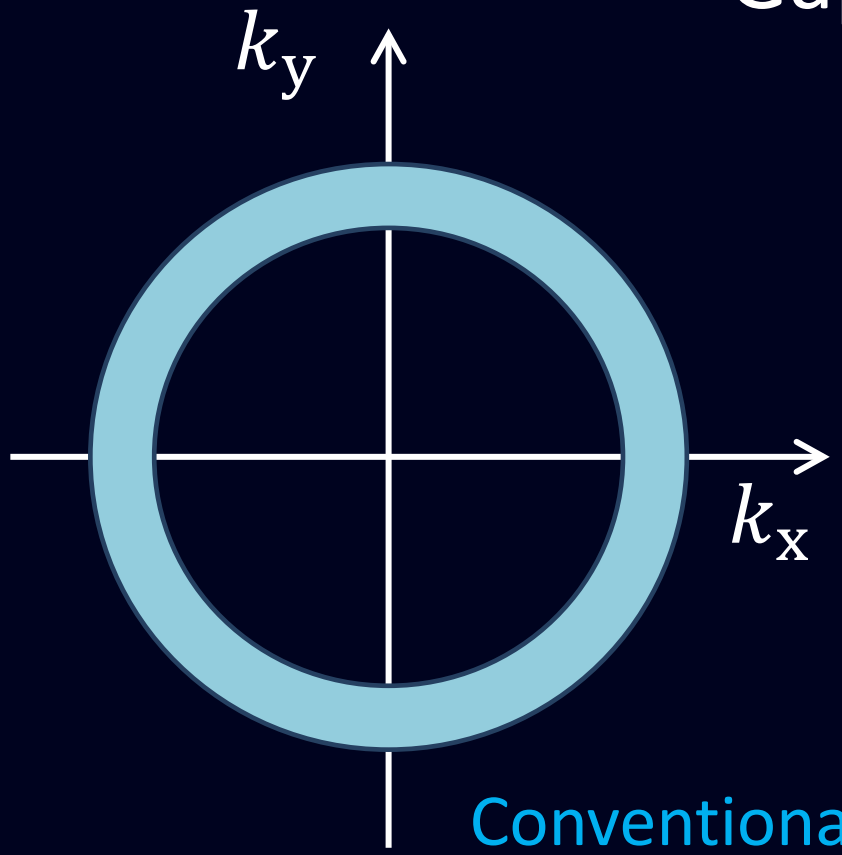

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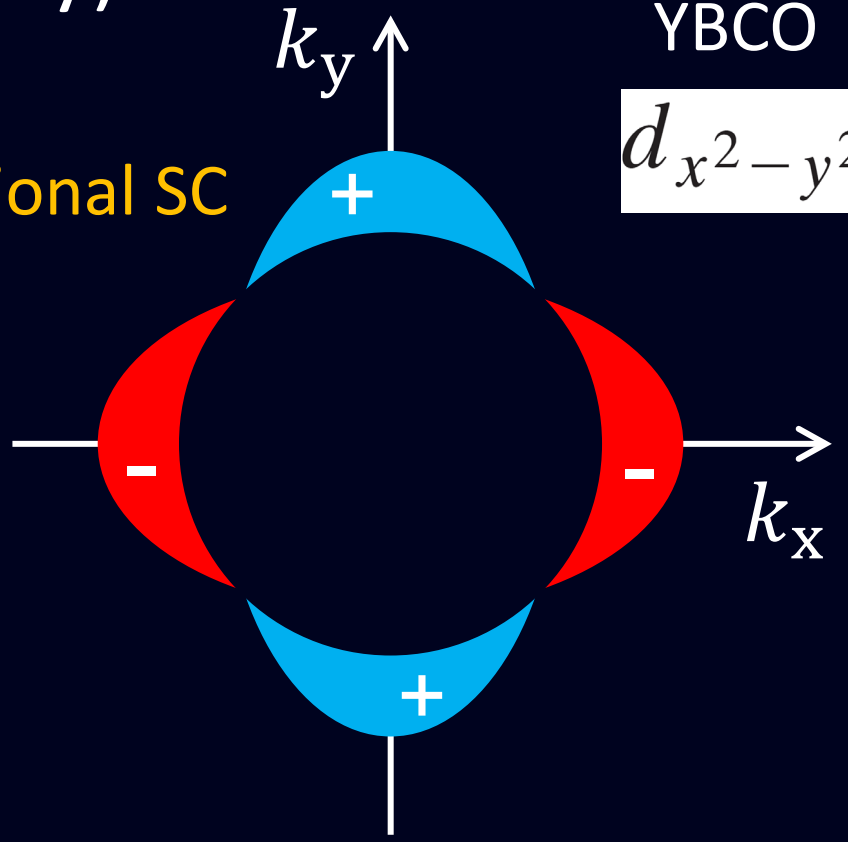
Unconventional SC

YBCO

$$d_{x^2-y^2}$$



Gap is k-***dependent*** (can have nodes)

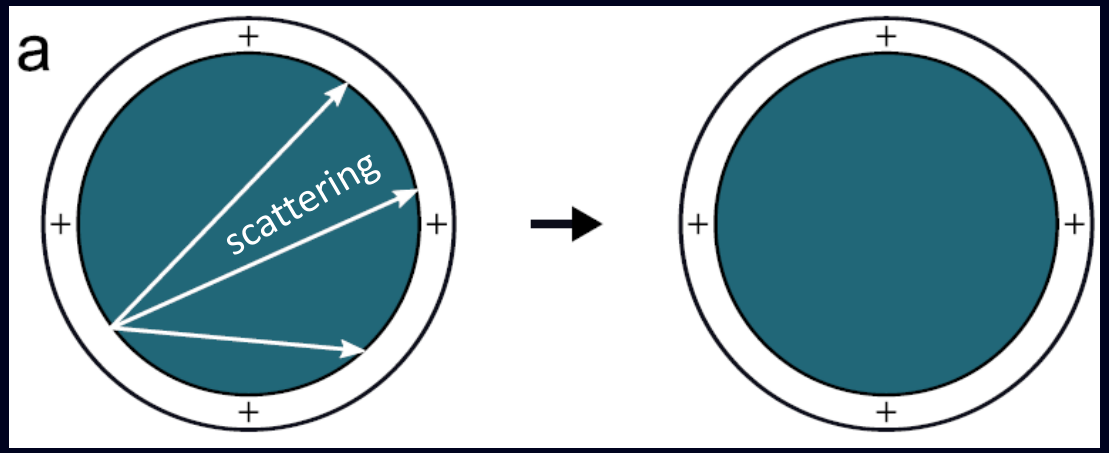
What's a good def for unconv SC?

$$\sum_{\mathbf{k}} \Delta(\mathbf{k}) = 0$$

Consequences of k-dependent Δ for unconv SCs: Elastic scattering

Mixes k-values

S-wave (conventional)



$$\Delta(k) = \Delta(-k)$$

$\Delta(\mathrm{k})$ has the same phase (sign) for all k-values

Gap is robust against elastic scattering

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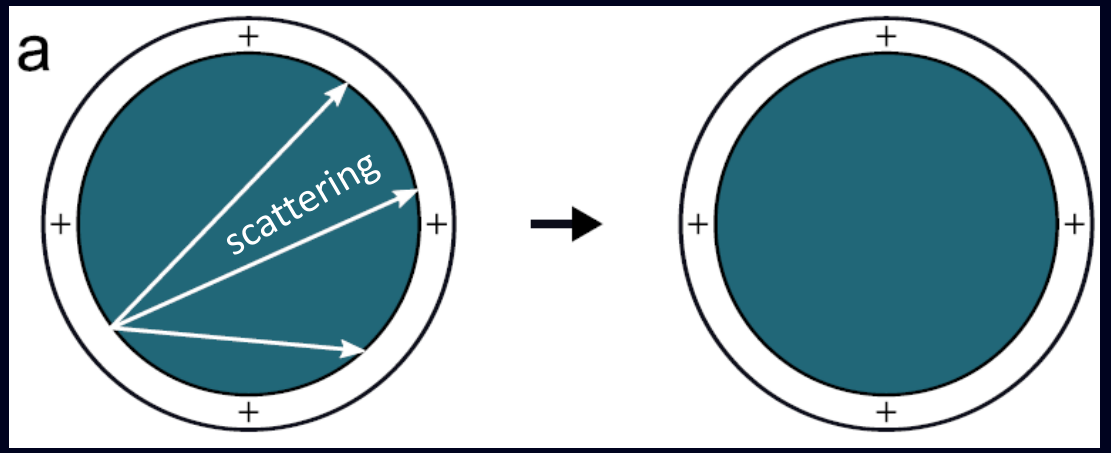

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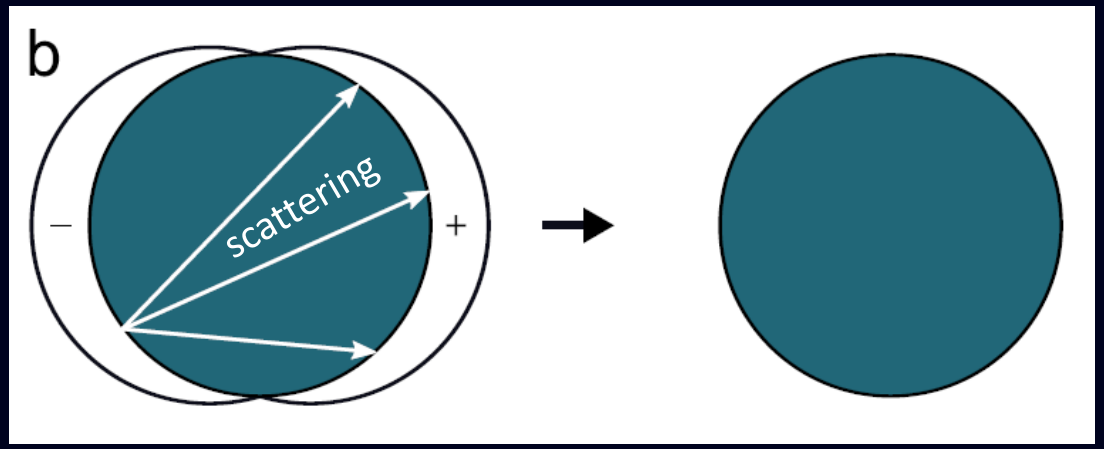
p-wave (unconventional)



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Order parameter cancels out under strong k-space averaging (gap completely destroyed)

Non s-wave SCs are generally sensitive to impurity scattering (need clean samples)

Consequences of k-dependent Δ for unconv SCs: Elastic scattering

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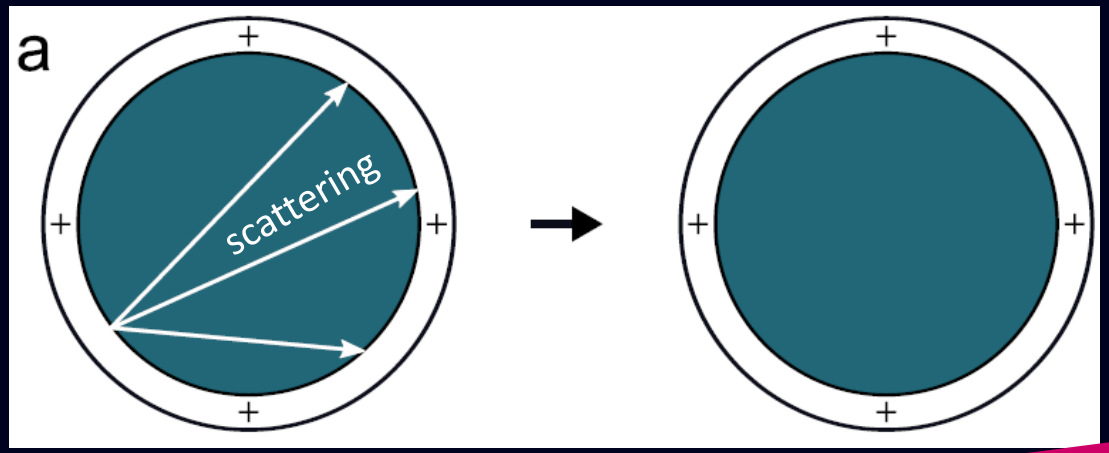


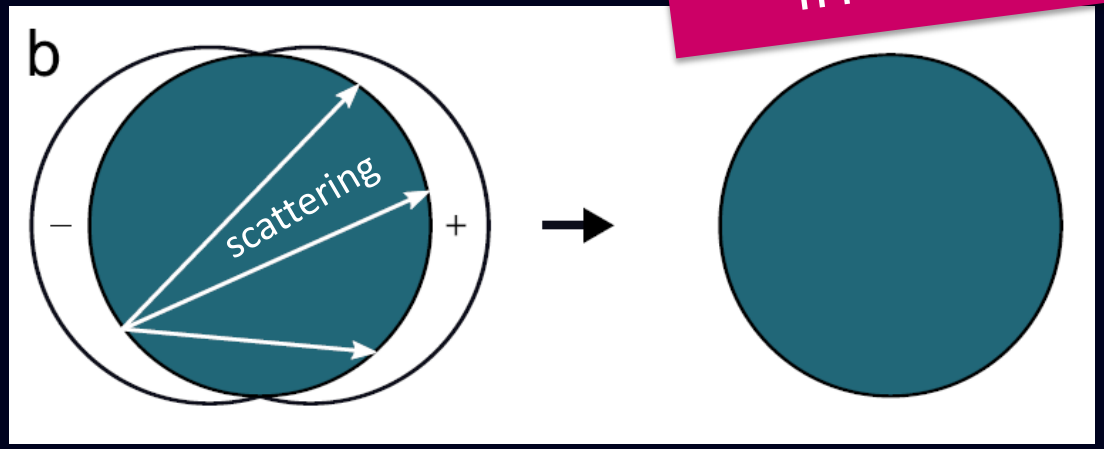
$$\Delta(k) = \Delta(-k)$$

$\Delta(\mathrm{k})$ has the same phase (sign) for all k-values

Gap is robust against elastic scattering

An unconventional gap is suppressed in the "dirty limit", where $l_{mfp} < \xi$

p-wave (unconventional)



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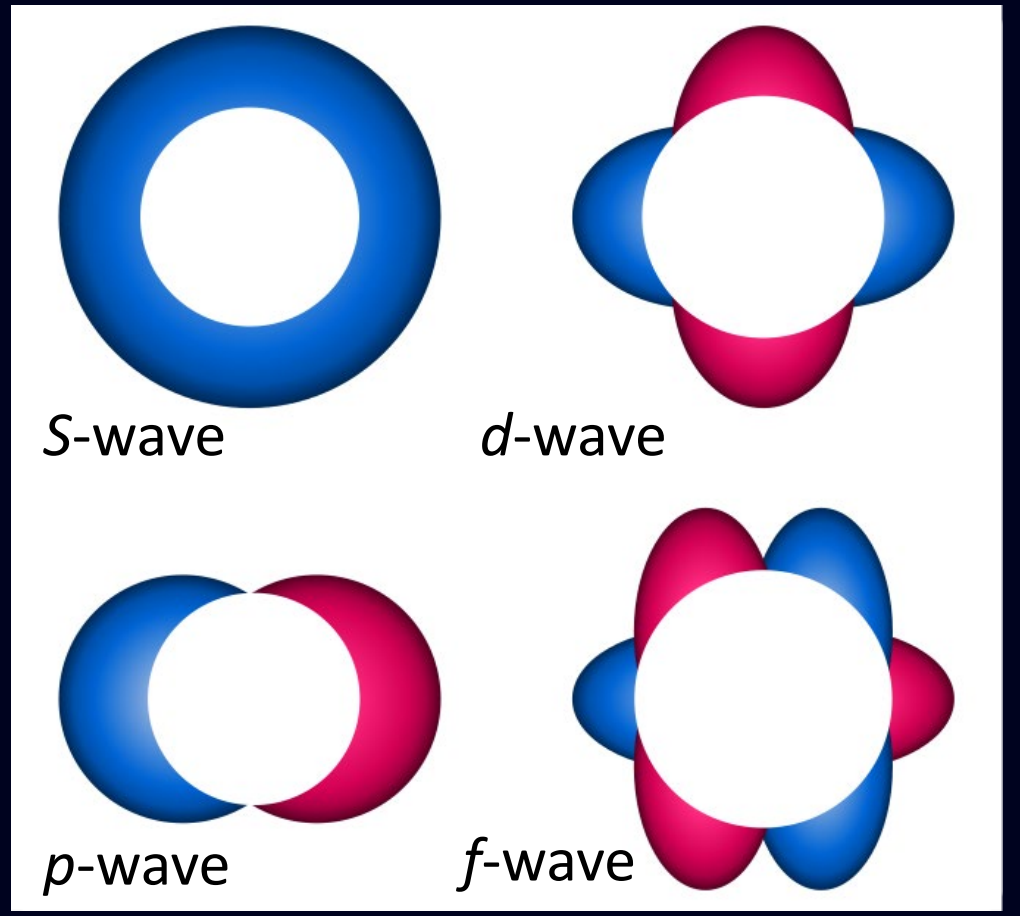
Pairing Symmetry:

$$\Psi\left(\sigma_{1,2}\,;\ k_{1,2}\,;\ \blacksquare\right)$$

Momentum (space)

Even

Odd



Pairing Symmetry:

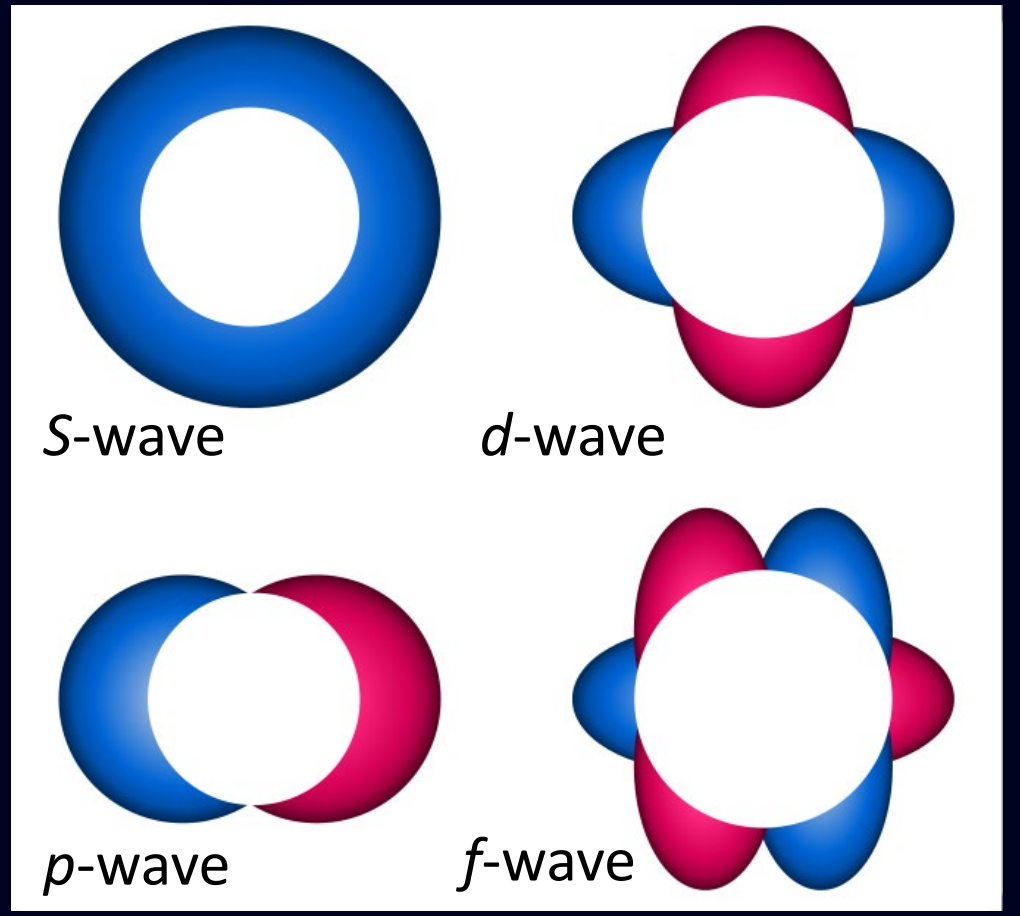
$$\Psi\left(\sigma_{1,2}\ ;\ k_{1,2}\ ;\ \omega_{1,2}\right)$$

Momentum (space)

Frequency (time)

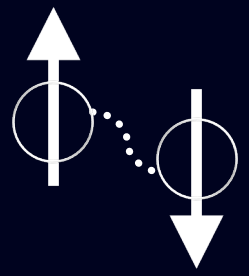
Even

Odd

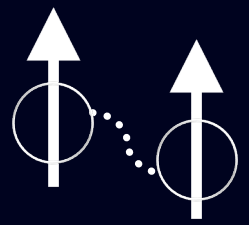


Allowed pairing symmetries

$$\Psi\left(\sigma_{1,2}\,;\,k_{1,2}\,;\,\omega_{1,2}\right)$$



Spin	Frequency	Momentum	
Singlet (odd) $\uparrow\downarrow - \downarrow\uparrow$	Even	Even	*s*, *d*
	Odd	Odd	*p*, *f*
Triplet (even) $\uparrow\downarrow + \downarrow\uparrow$, $\uparrow\uparrow$, $\downarrow\downarrow$	Even	Odd	*p*, *f*
	Odd	Even	*s*, *d*



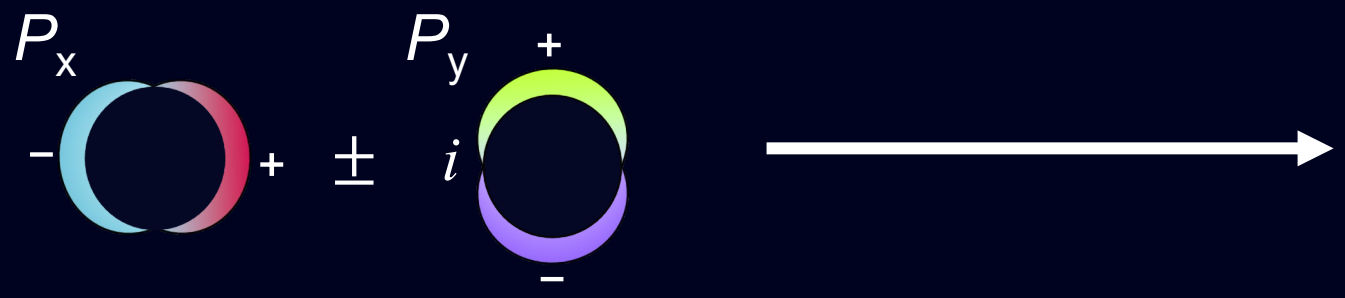
S-wave: Nb, Al, MoGe,…
d-wave: Cuprates (e.g. YBCO)

^{3}He, UPt_3

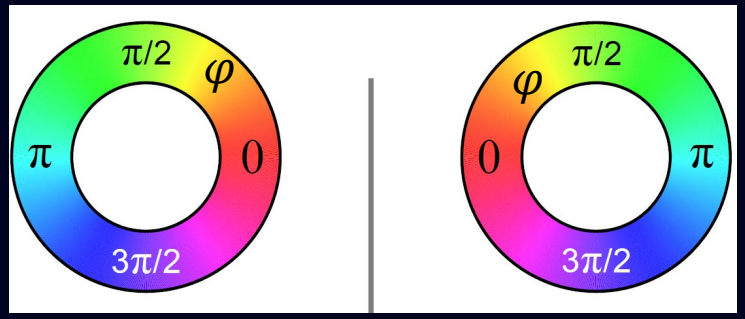
So far, only observed in S-F hybrids (via proximity effect)

Hybrid gaps

A gap can also be a hybrid combination of different symmetries



Example: Chiral *p*-wave



$P_x + i P_y$ $P_x - i P_y$

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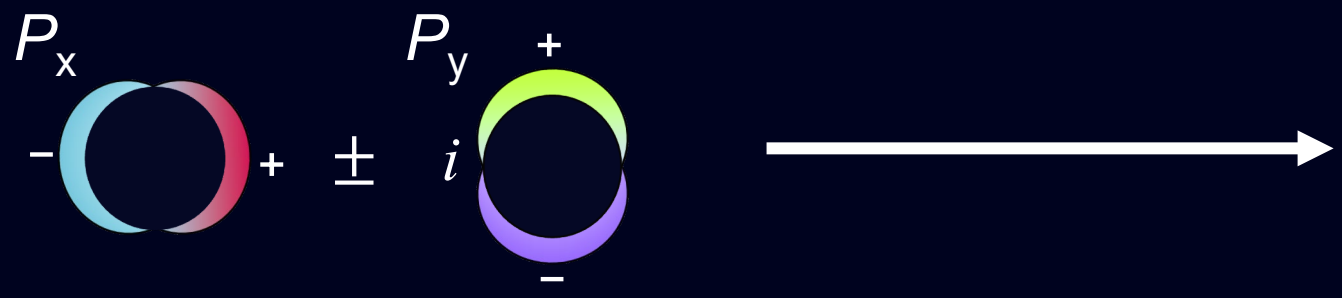

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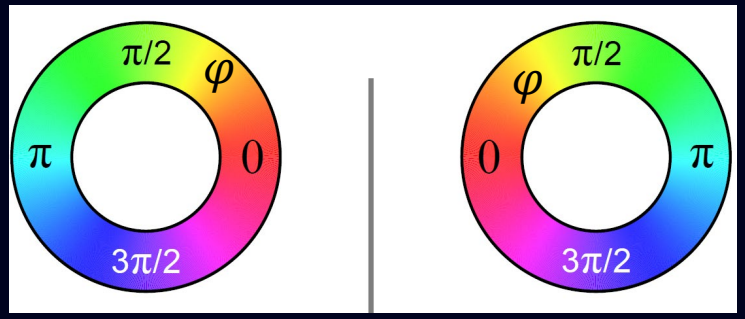

$P_x + i\,P_y$ $P_x - i\,P_y$

Phase is continuously winding around the gap

$$\Psi = |\Psi|\ e^{i\varphi}$$

Chirality → phase winding has a directions

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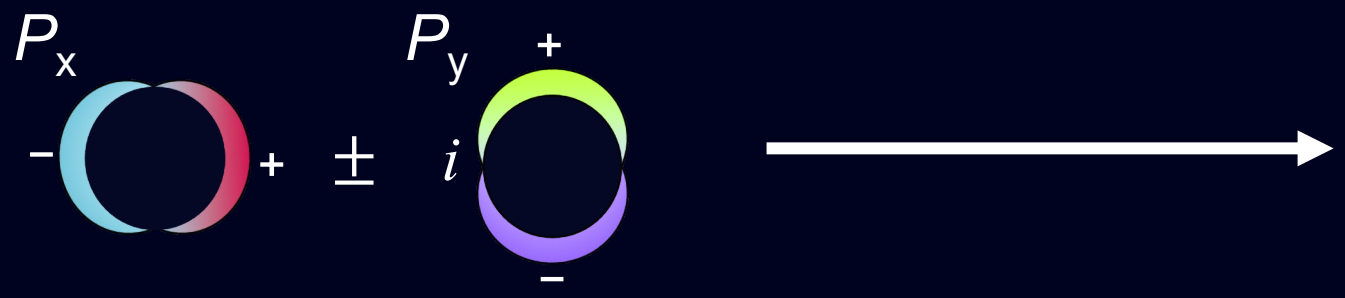

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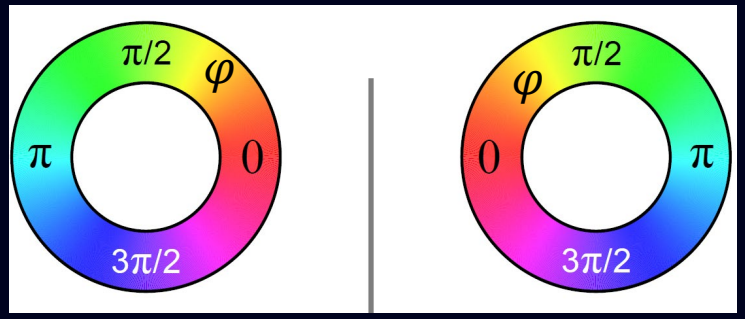

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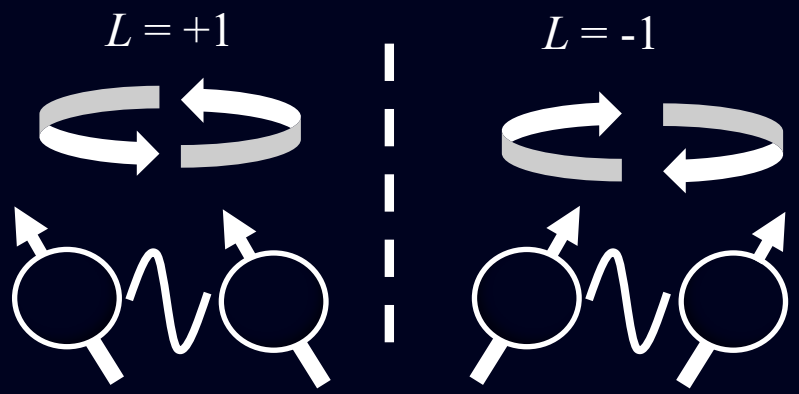
What does phase winding produce again?



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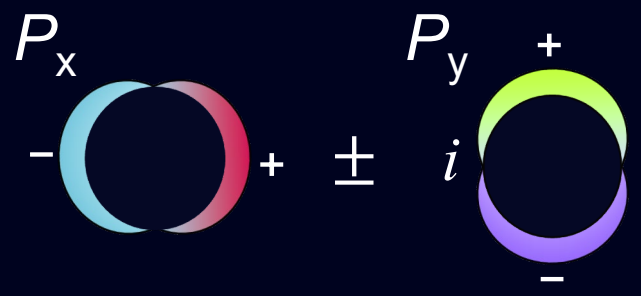

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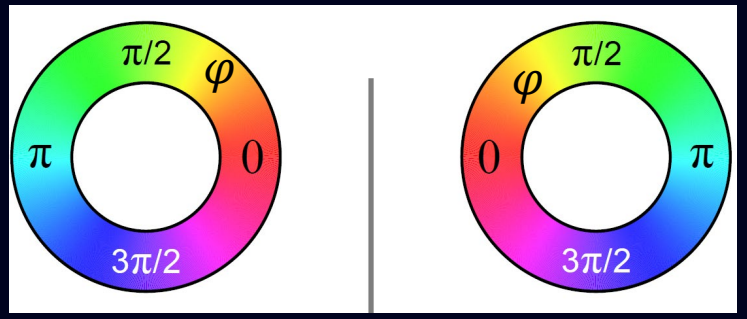

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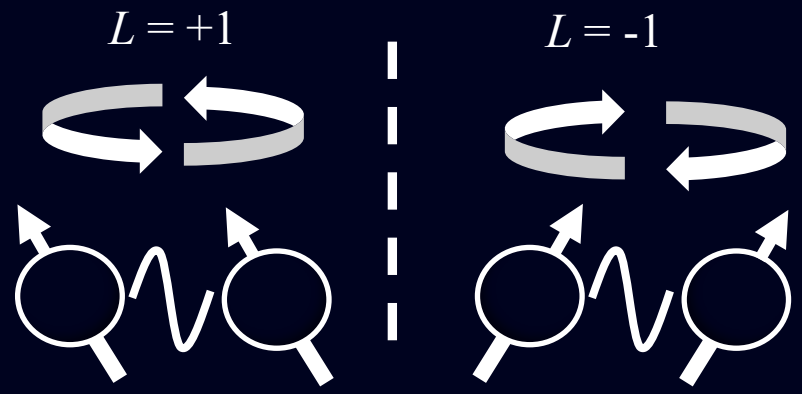

Chiral superconductor?

Hybrid gaps

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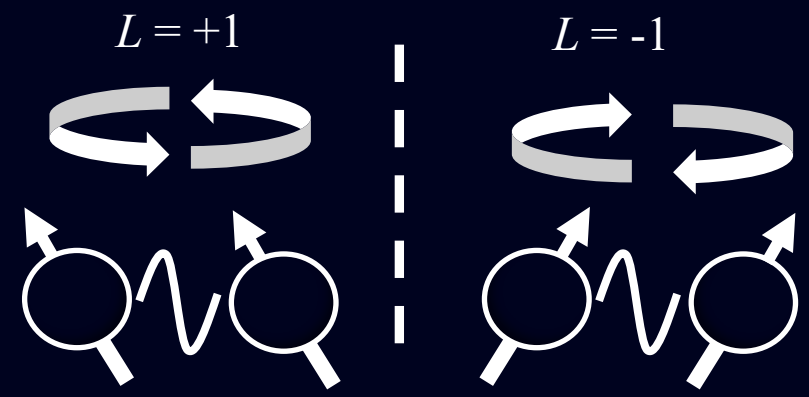
Favourite quote:

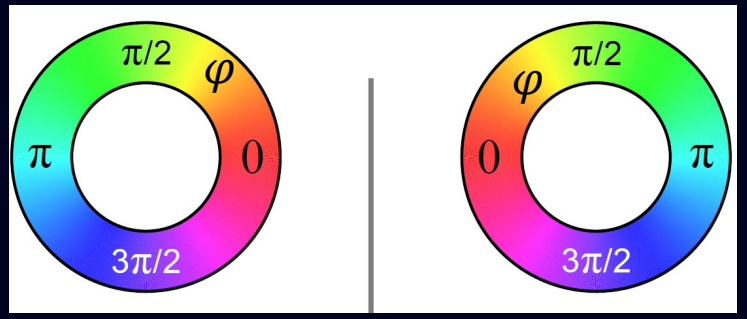
"Every potato is chiral. It doesn't mean it's interesting"

Michel Orrit

What does phase winding produce again?



Example: Chiral *p*-wave



$P_x + i P_y$ $P_x - i P_y$

Phase is continuously winding around the gap

$$\Psi = |\Psi| \, e^{i\varphi}$$

Chirality → phase winding has a directions

Chiral superconductor?

End of Lecture 3

REVIEWS OF MODERN PHYSICS, VOLUME 75, APRIL 2003

The superconductivity of Sr_2RuO_4 and the physics of spin-triplet pairing

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A lot of the material on symmetries can be found in
1. 2003 review by Mackenzie & Maeno
2. Kaveh Lahabi's PhD thesis: Chapter 2
(click here for the link) or scan the QR.

Kaveh Lahabi (2025)