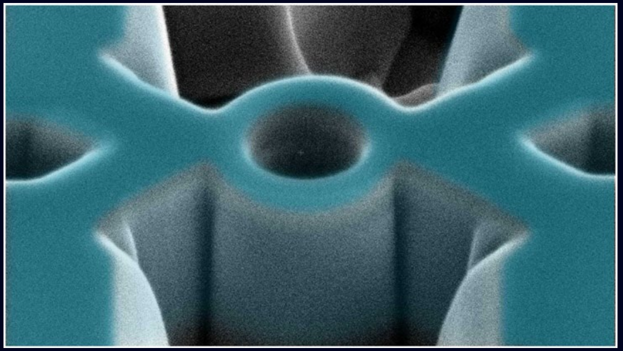
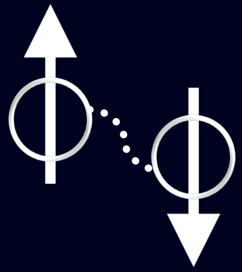
Superconductivity: Lecture 2

Mesoscopic superconductors & phase diagrams

Ψ () $= \Psi$ ()

Kaveh Lahabi (2025)

Recap

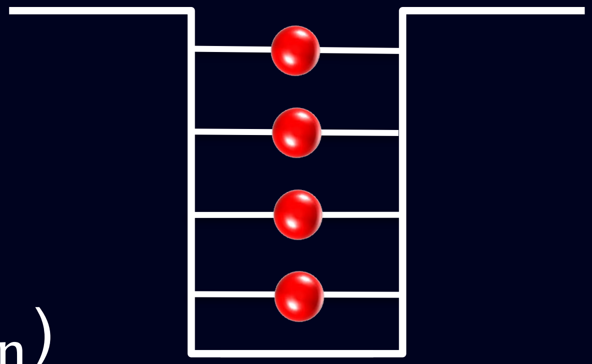
Normal materials: electrons (**fermions**)

Ψ depends on the number of particles.

In a typical macroscopic system:

- Ψ unknown
- QM coherence destroyed by scattering

$$\Psi\ (e_1, e_2, e_3, \ldots, e_n)$$

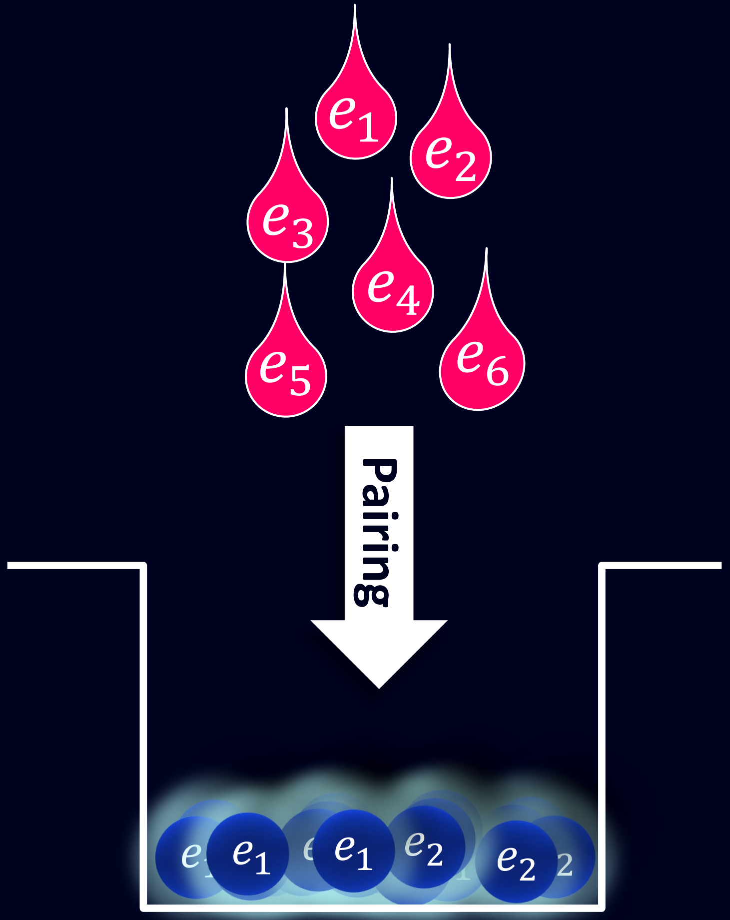


Superconductors: **Bosonic condensates**

Cooper pairs live in a *one coherent macroscopic state*

Macroscopic wavefunction = same as a single e pair

$$\Psi\ (e_1, e_2)$$

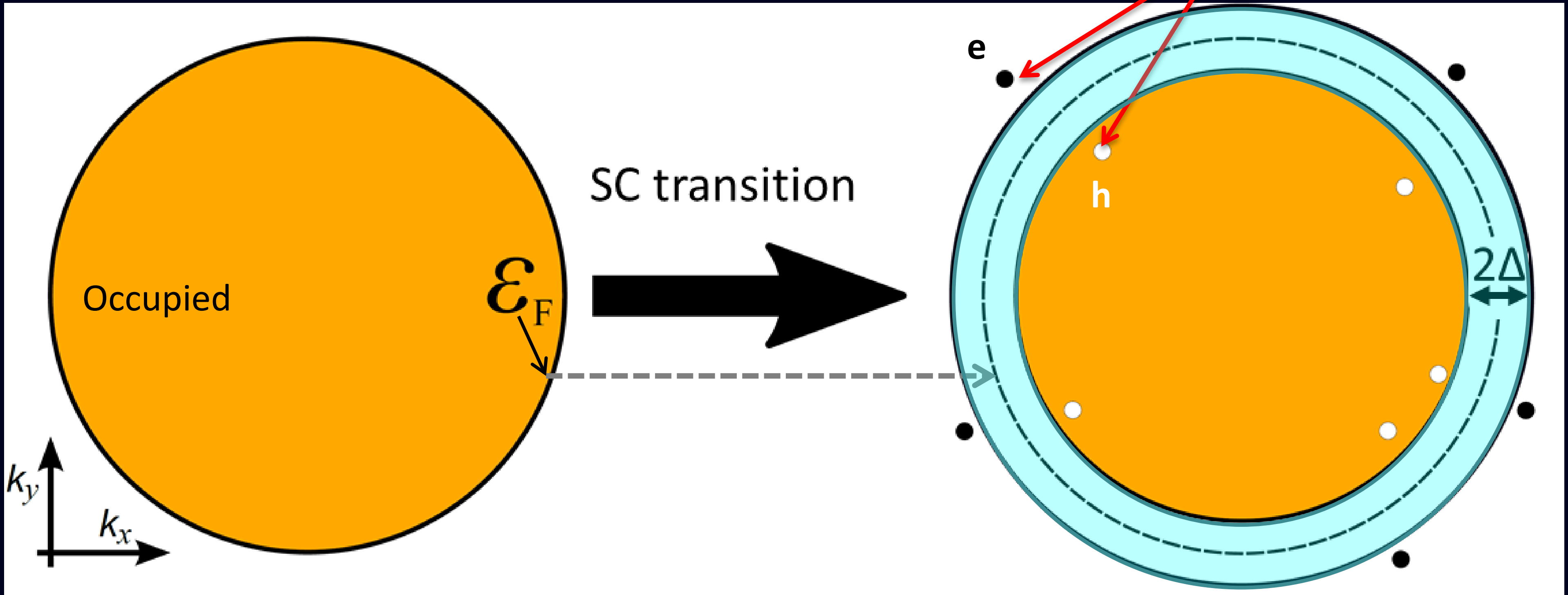


Don’t think of a SC as a collection of separate pairs/particles

Paired electrons are now a single macroscopic entity

‘Down with the fermions! Long live the bosonic condensate!’

Electrons/holes can be excited above/below the gap, but they can’t exist within it.



Normal (T>Tc)

SC (T<Tc)

Elegance of superconductors: Macroscopic wavefunction

Schrödinger

$$\frac{1}{2m}\left(-\imath\hbar\boldsymbol{\nabla} - e\boldsymbol{A}\right)^2\psi + U\psi = E\psi$$

$$\Psi = |\Psi(\mathrm{r})|e^{i\varphi(r)}$$

Ginzburg-Landau equations

$$\frac{1}{2m^\star}\left(-\imath\hbar\boldsymbol{\nabla} - e^\star\boldsymbol{A}\right)^2\psi_s + \beta|\psi_s|^2\psi_s = -\alpha\psi_s$$

$$-\alpha \leftrightarrow E$$

$$-\alpha = \frac{\hbar^2}{2m^\star\ \xi^2(T)}$$

$$\boldsymbol{\nabla} \times \boldsymbol{A}\ \text{(vector potential)} = \mu_0\boldsymbol{h}\ \text{(magnetic field)}$$

$$e^* = 2e$$

$$m^* = 2m_e$$

$$\xi(T) = \frac{\xi(0)}{\sqrt{1-\frac{T}{T_{c0}}}}$$

$$\boldsymbol{j} = \boldsymbol{\nabla}\times\boldsymbol{h} = \frac{e^\star}{2m^\star}\left[\psi_s^\star(-\imath\hbar\boldsymbol{\nabla} - e^\star\boldsymbol{A})\psi_s + \psi_s(\imath\hbar\boldsymbol{\nabla} - e^\star\boldsymbol{A})\psi_s^\star\right]$$

Once in the SC state, only two length scales matter: ξ and λ

$$\Psi = |\Psi(\mathrm{r})|e^{i\varphi(r)}$$

ξ : Coherence Length
"stiffness" of the amplitude

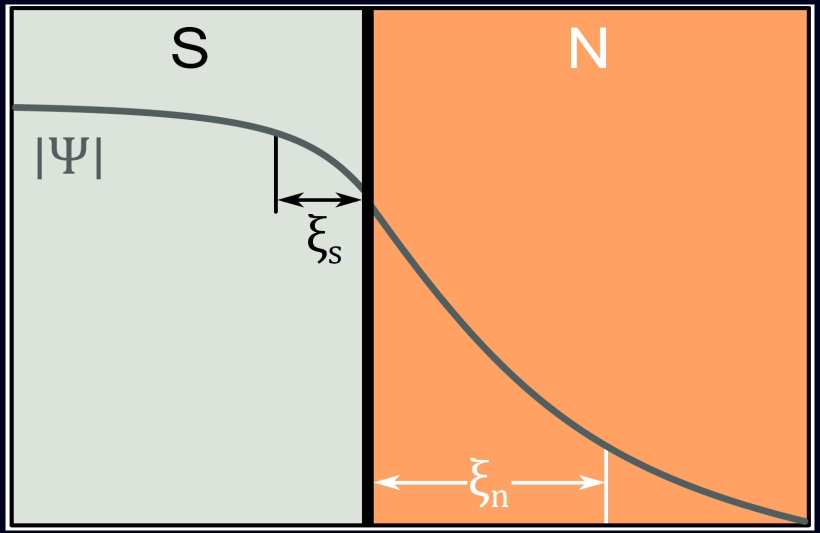
How rapidly does |Ψ| (Cooper pair density) "bend" in real space

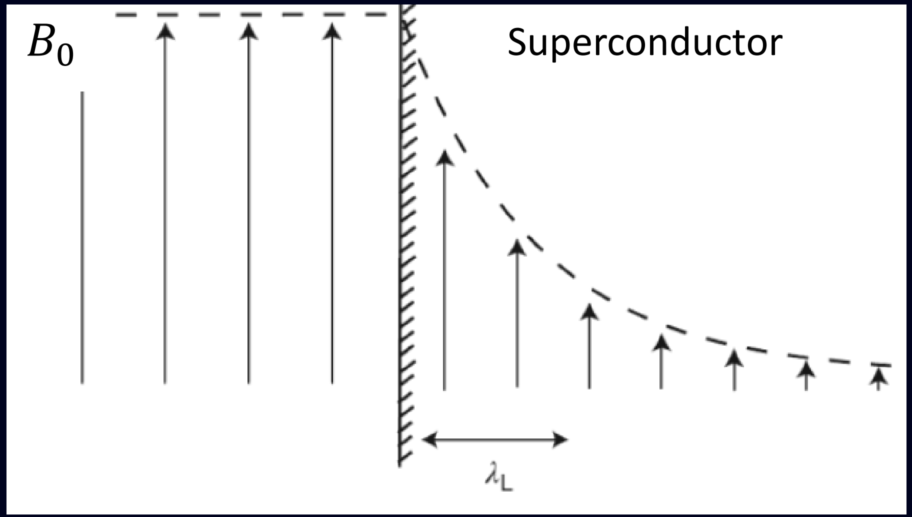
λ: magnetic penetration depth
(stiffness of the phase φ)

λ: Characteristic decay length of magnetic fields inside a SC

What's the link between φ and screening of magnetic field?

λ and ξ are <u>independent</u> material parameters.





Today's lecture

What happens when you place a superconductor in a magnetic field?

Why does a large enough magnetic field destroy superconductivity?
(why do superconductors have an upper critical field?)

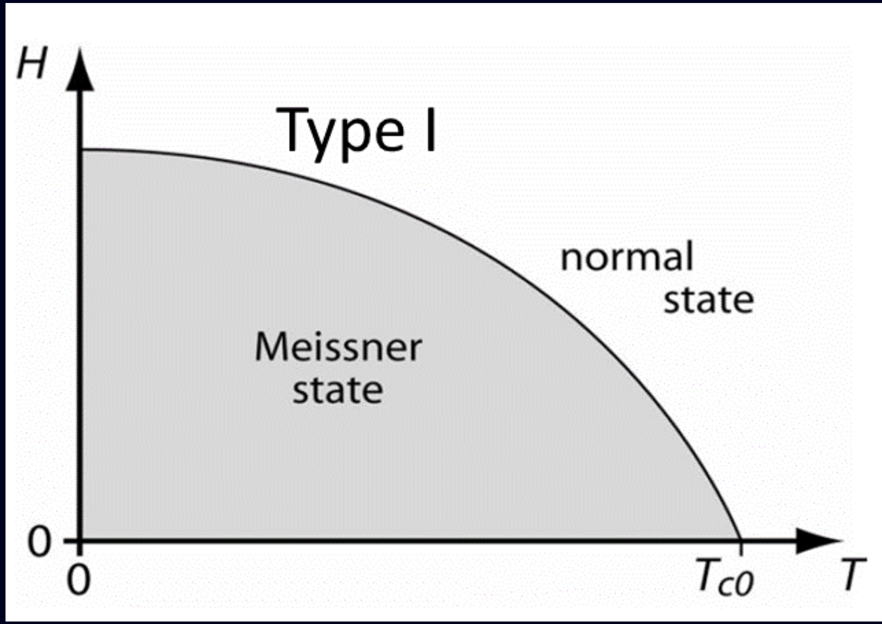
Why do we have two "types" of superconductors?

What's different about the magnetic field response of mesoscopic structures?

H-T phase diagrams

$$\Psi = |\Psi(\mathrm{r})| e^{i\varphi(r)}$$

How does magnetic field destroy superconductivity?



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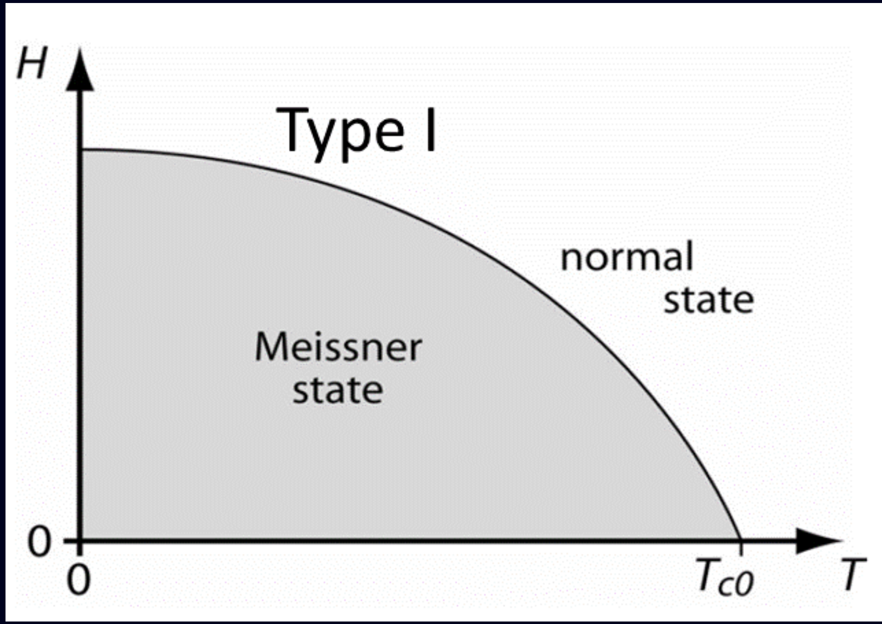
How does magnetic field destroy superconductivity?

Magnetic flux couples to the orbital phase (φ) of Ψ



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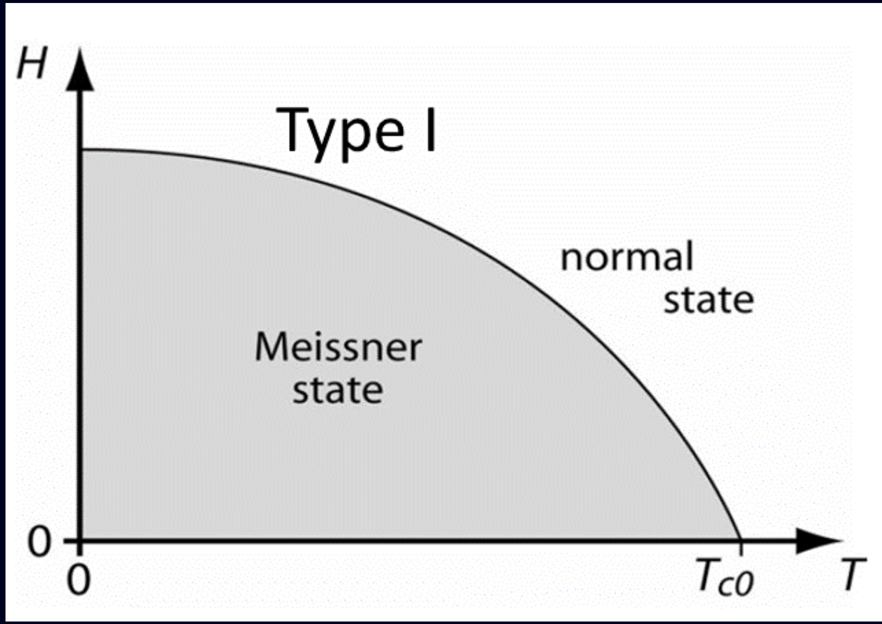

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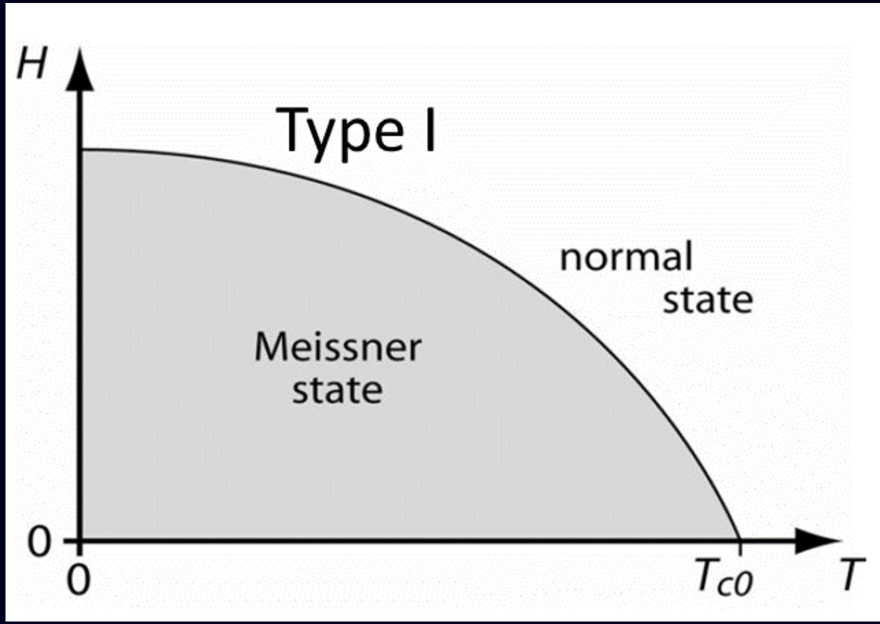
Supercurrent costs (kinetic) energy. Where does that come from?



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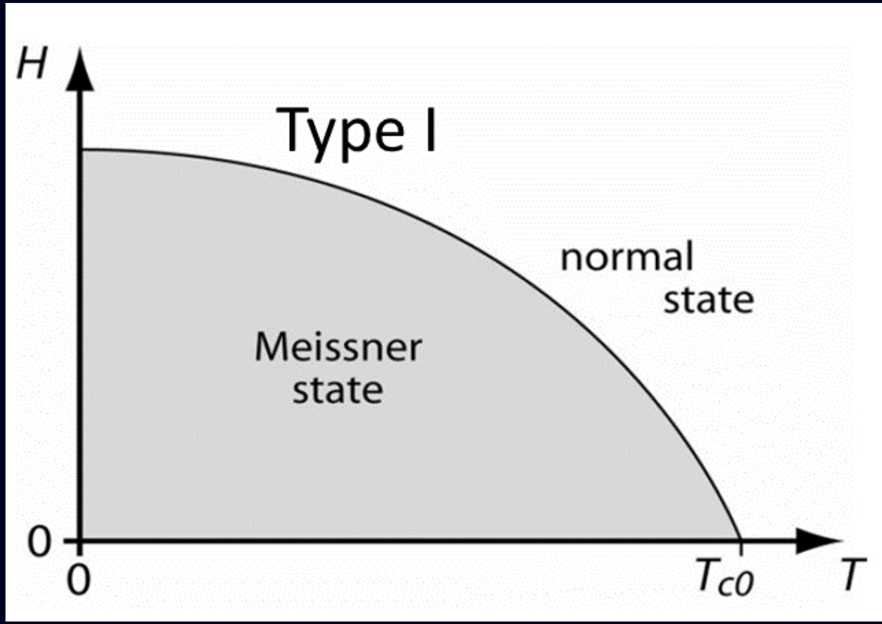

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Magnetic flux enters in quantized units of

$$\Phi_0 = \frac{h}{2e} \approx 2.067 \times 10^{-15}\ \mathrm{T/m^2}$$

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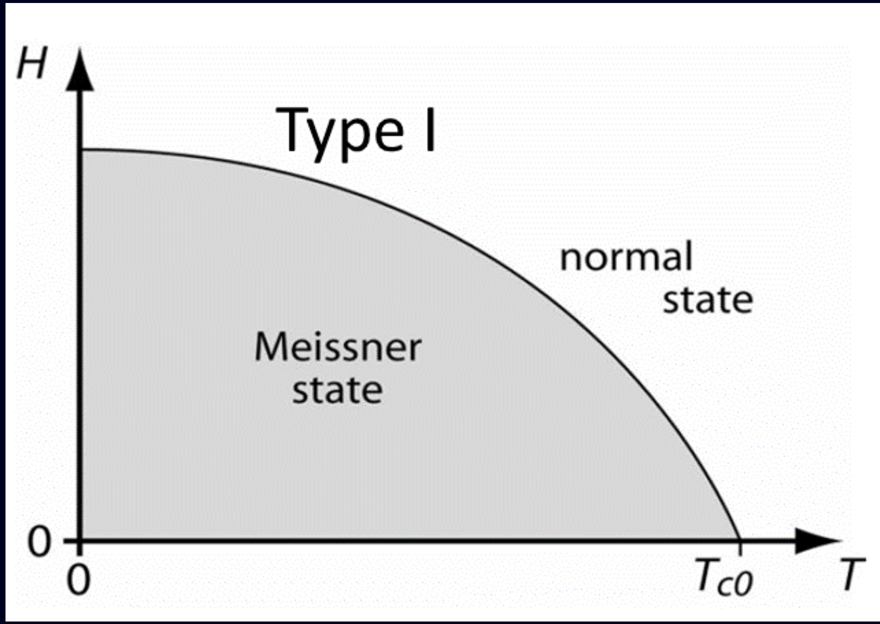

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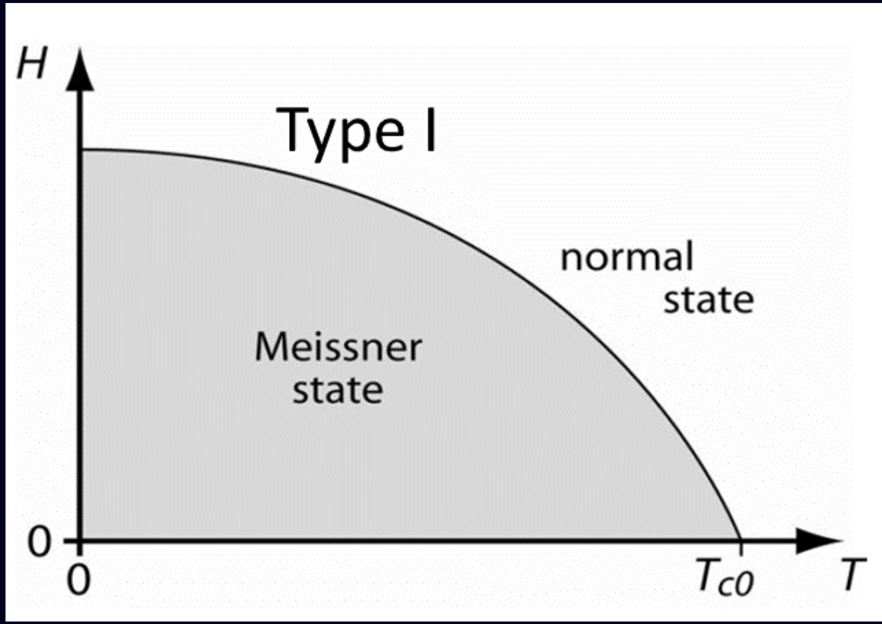

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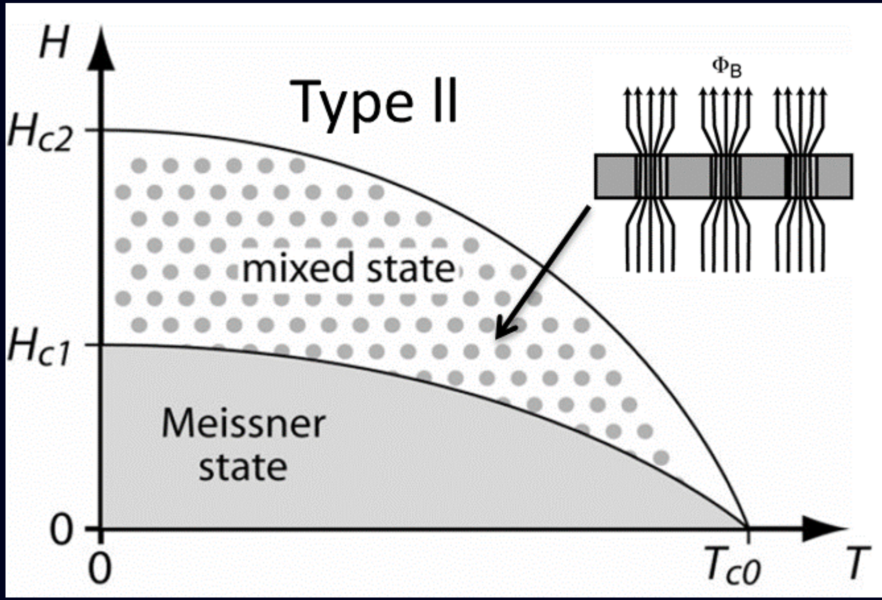

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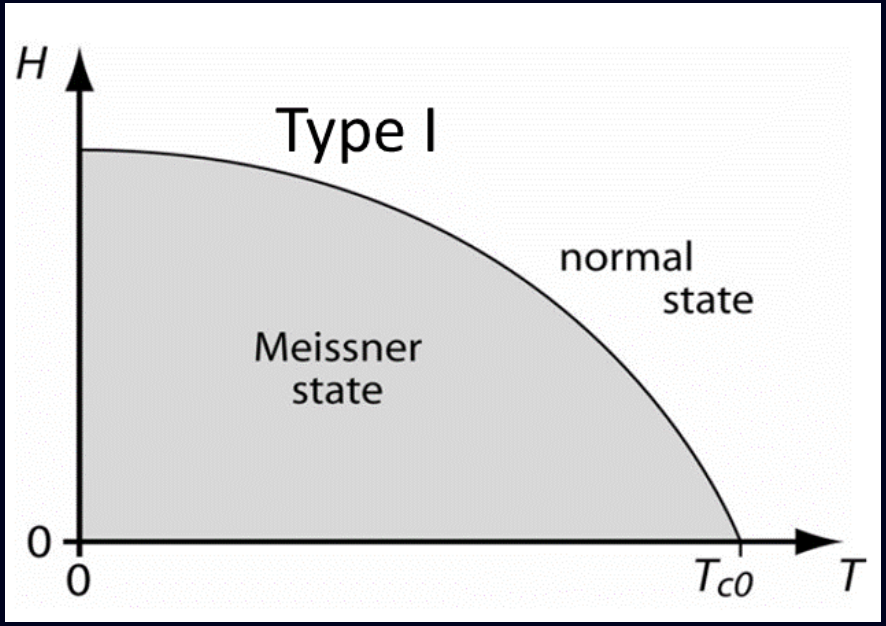

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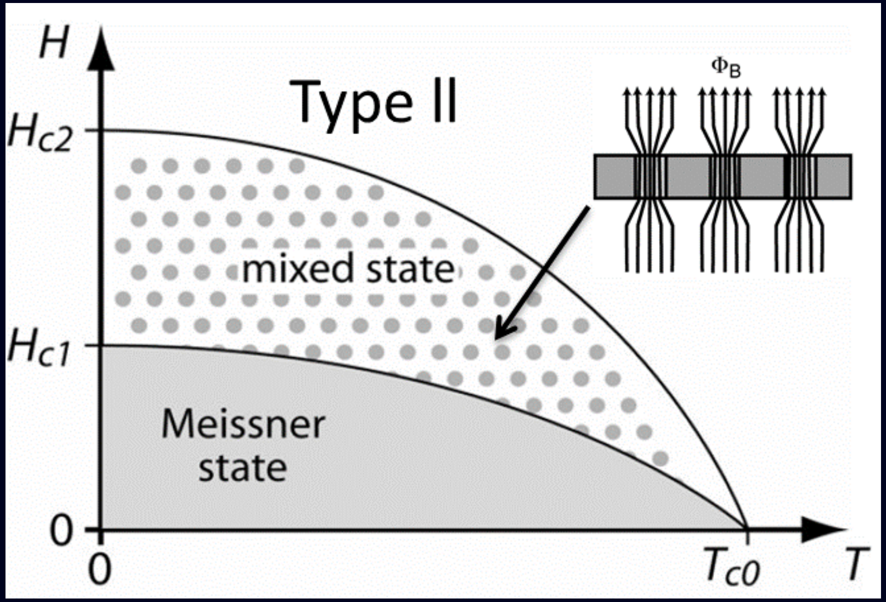
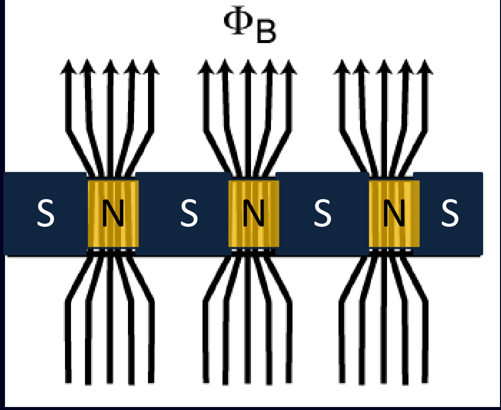
So, why two types?
Why some SCs host vortices and others don't?



Flux enters through the normal core in quantized units of Φ_0

φ winds by 2π around the flux, generating a circulating current $J \sim 1/r$
At $r > \lambda$: $J \rightarrow 0$, $B \rightarrow 0$



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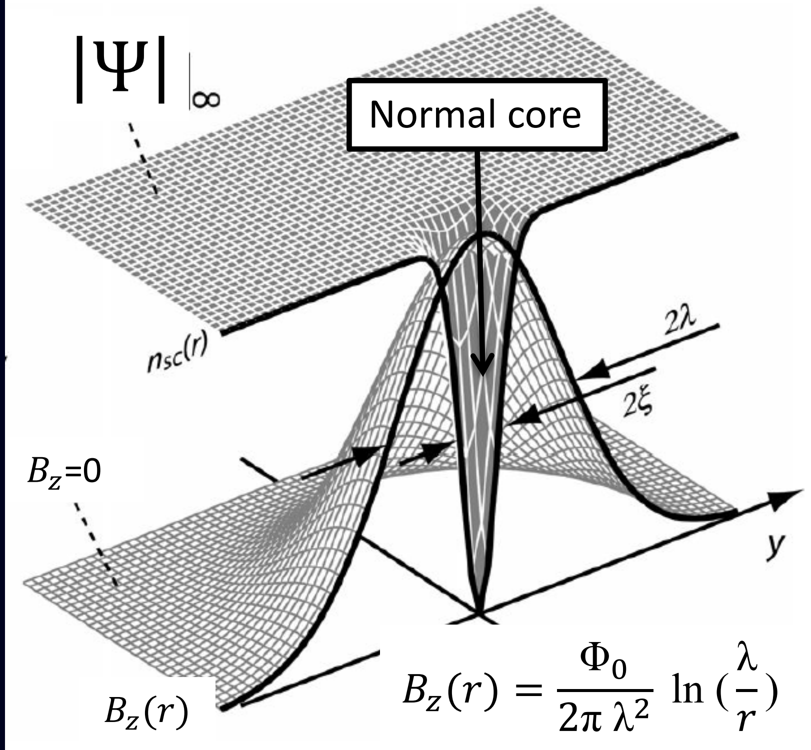

Type II: Anatomy of an Abrikosov Vortex

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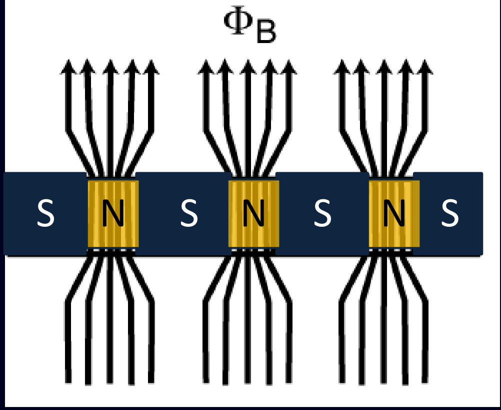

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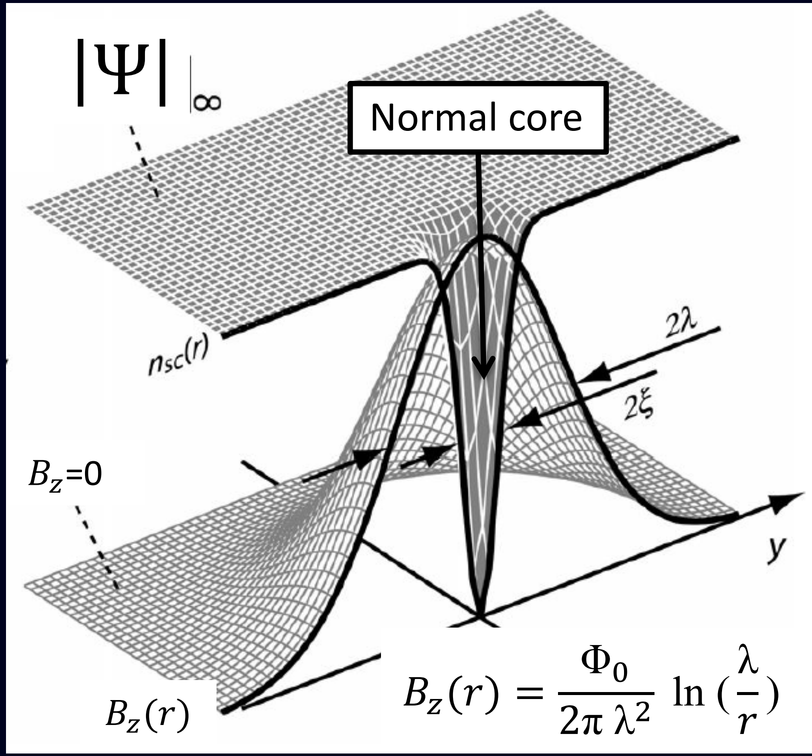

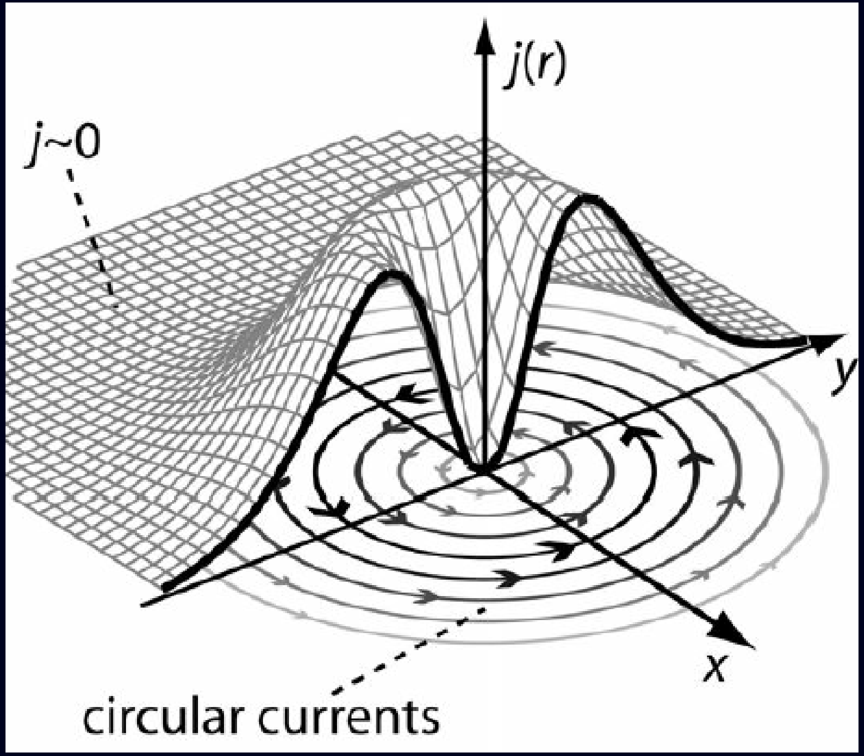

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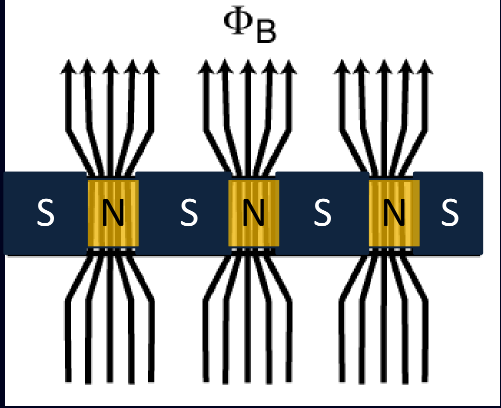

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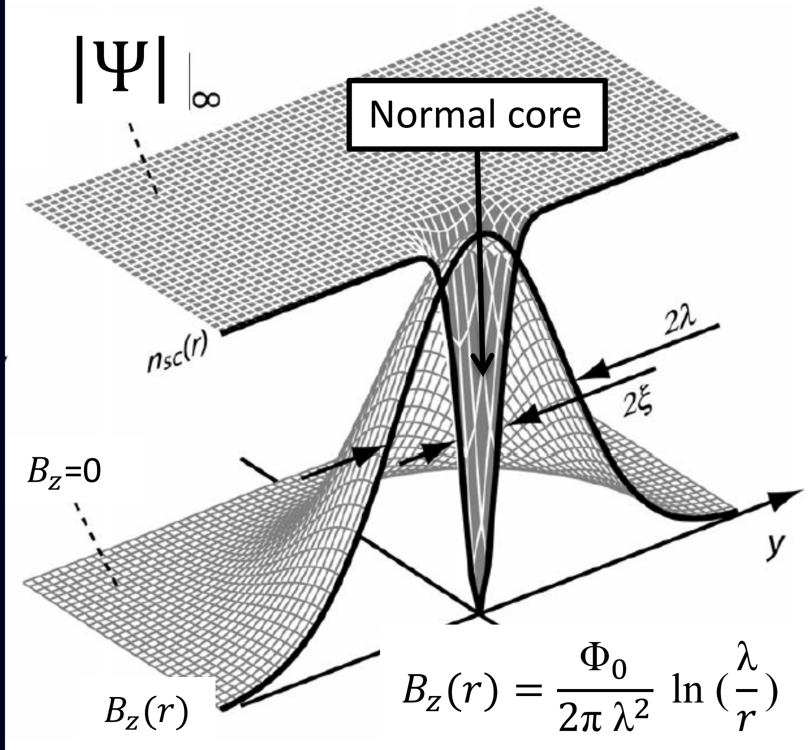

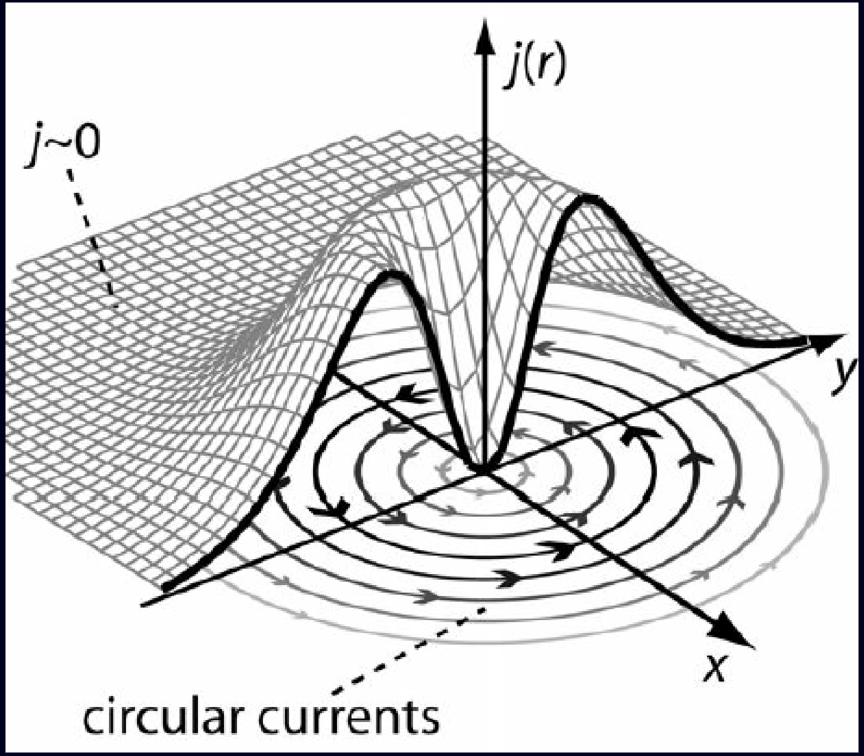

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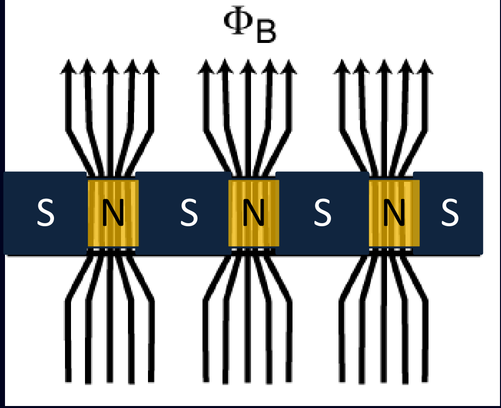

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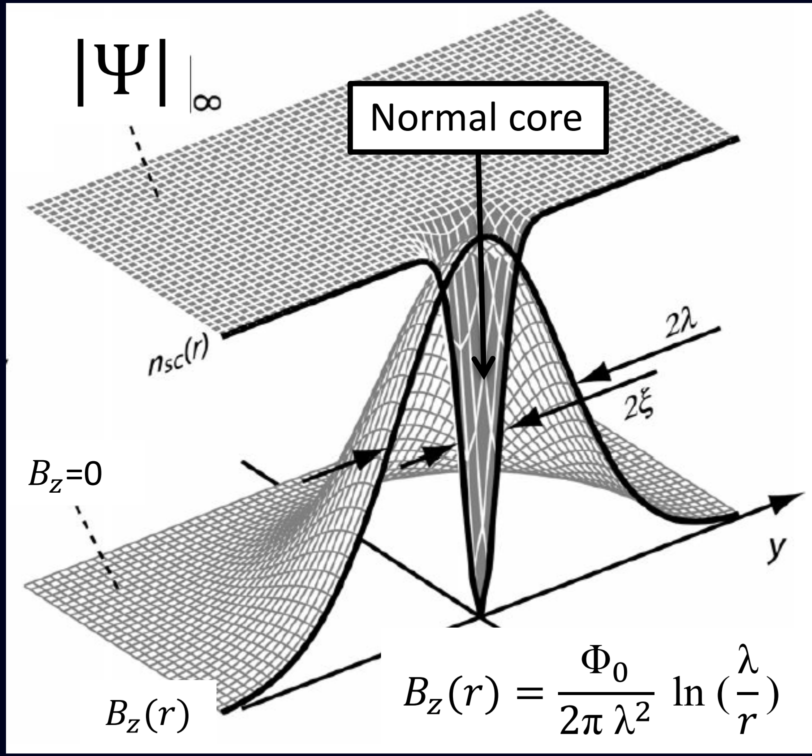

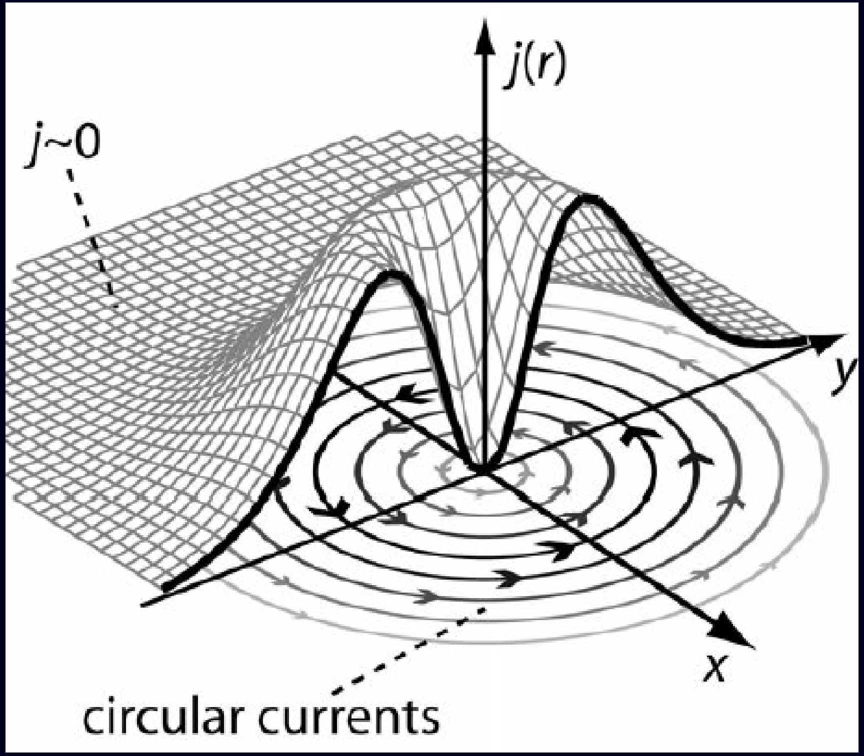

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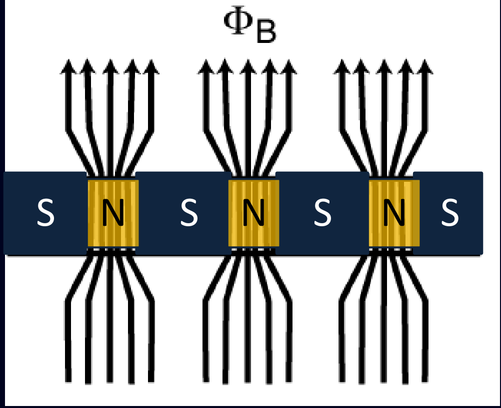

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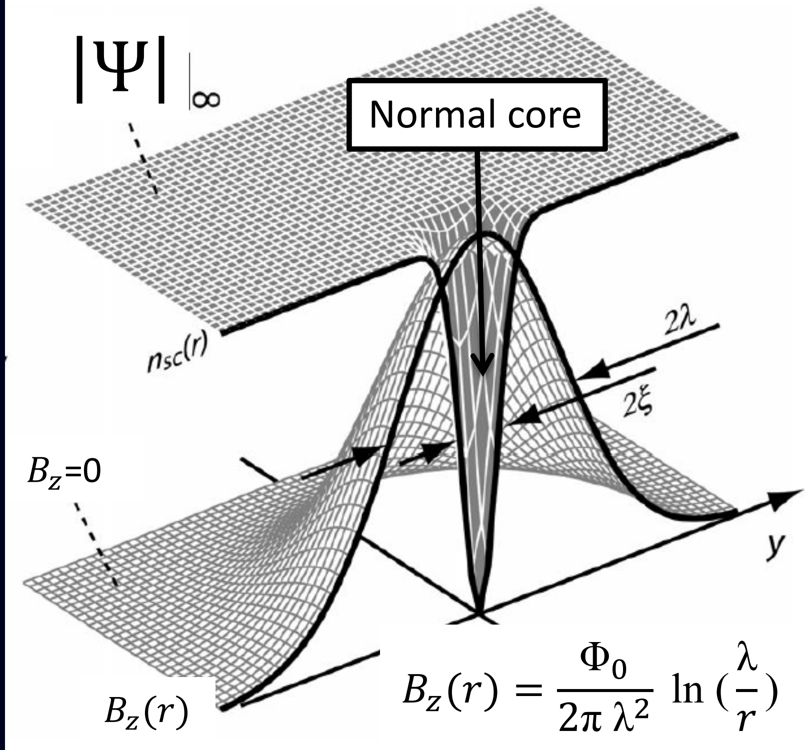

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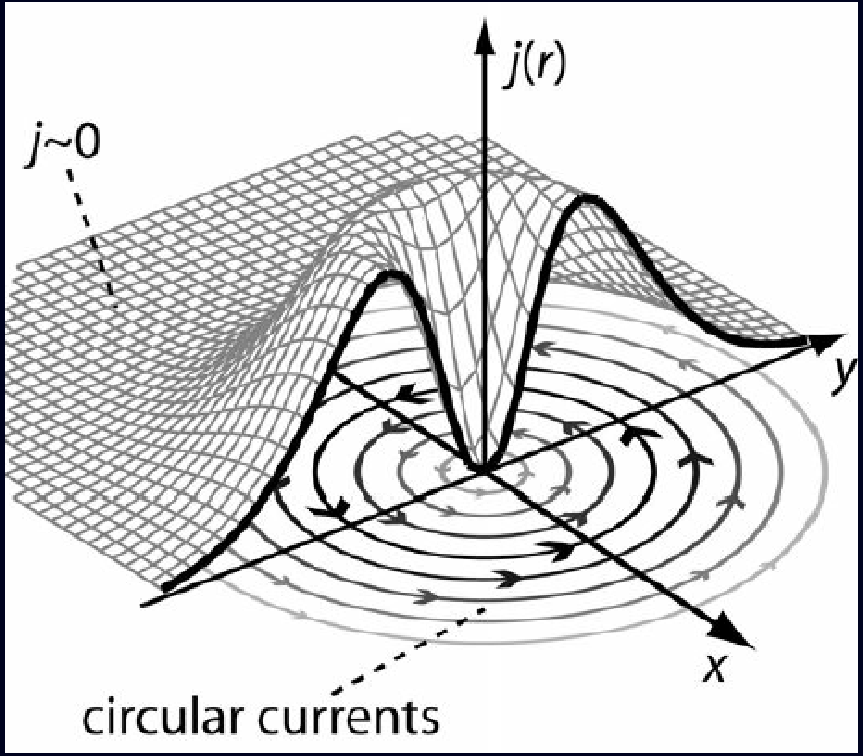
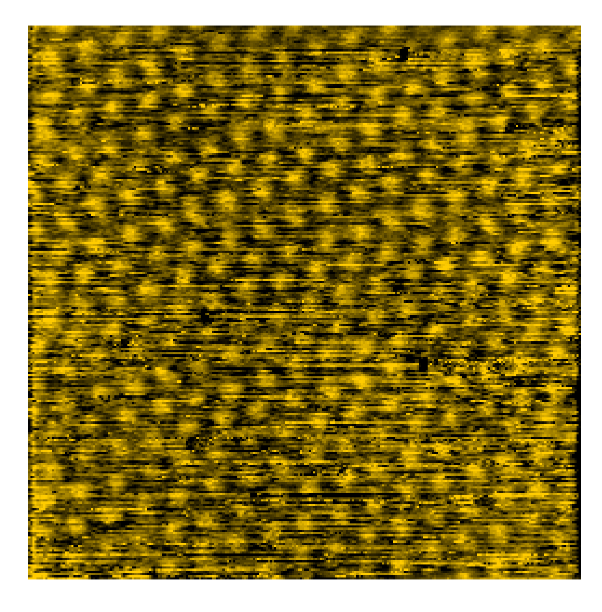
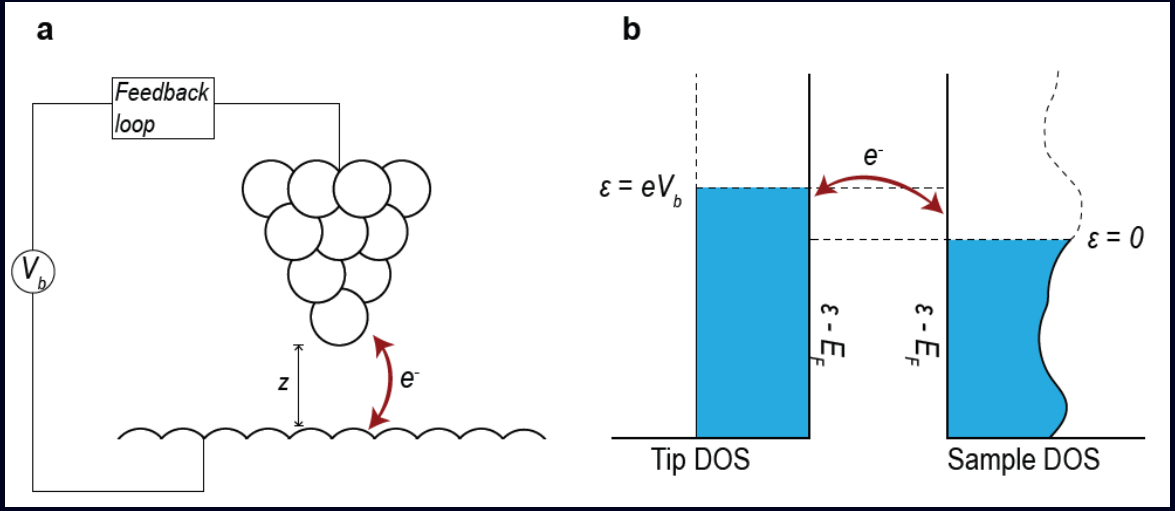


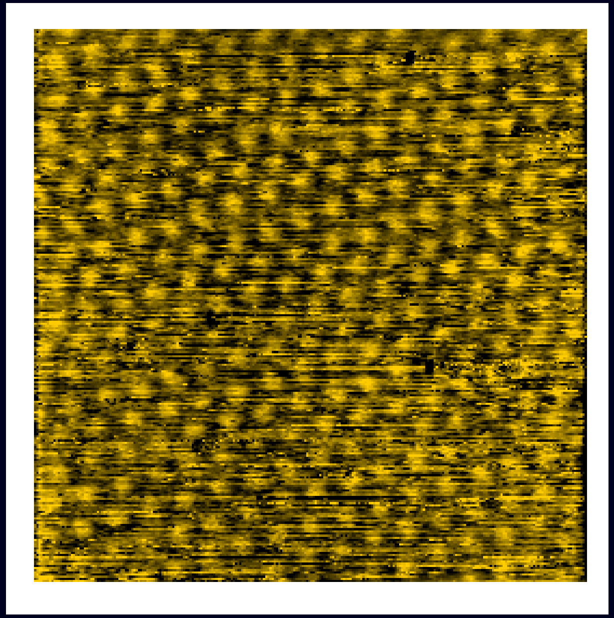
Abrikosov vortex lattice



How was this image taken?

Imaging vortices with STM: What does STM probe?





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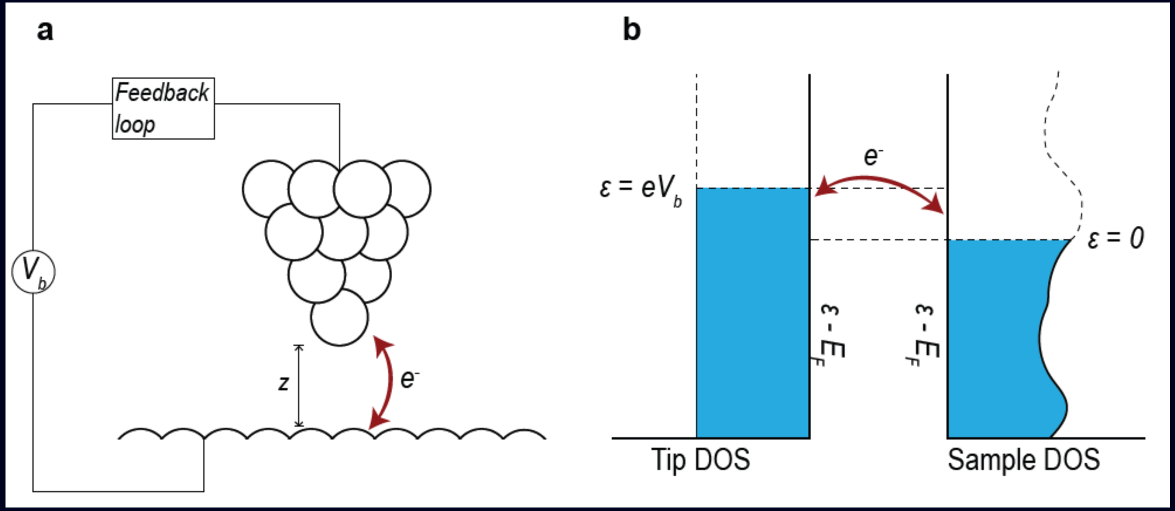
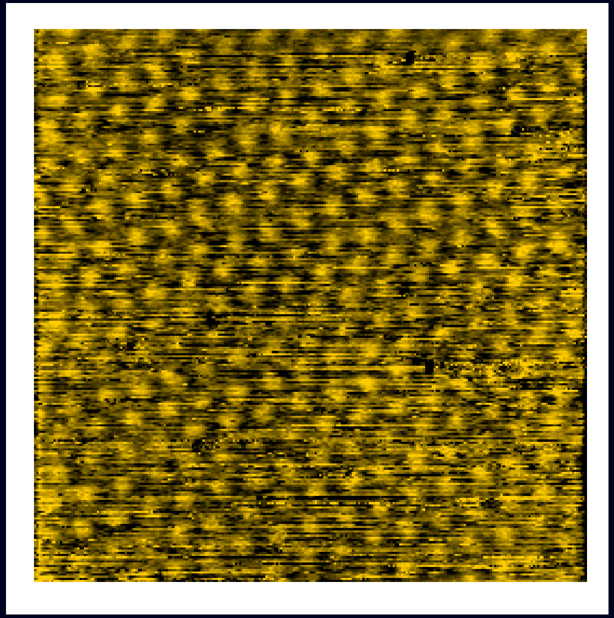
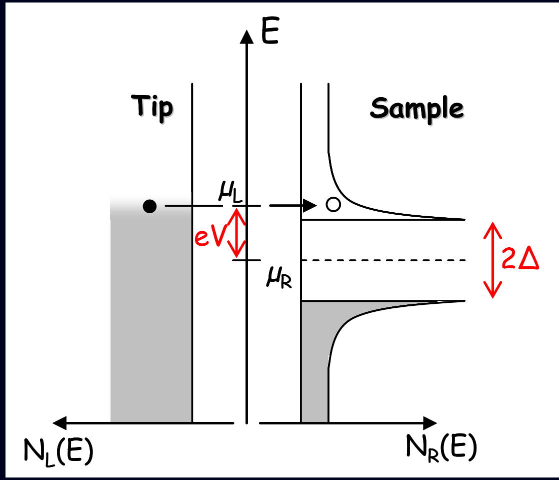
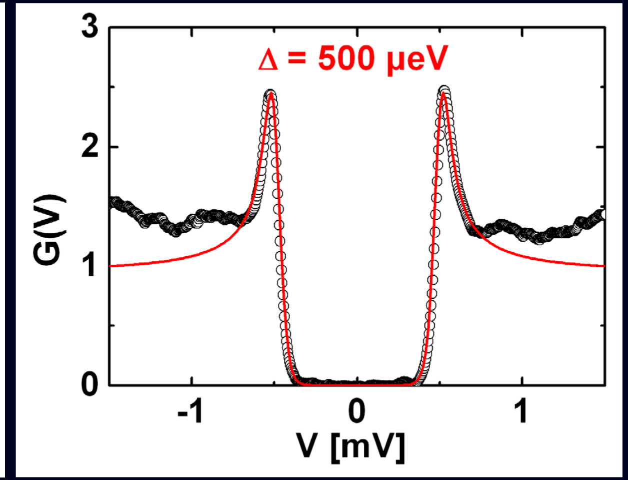


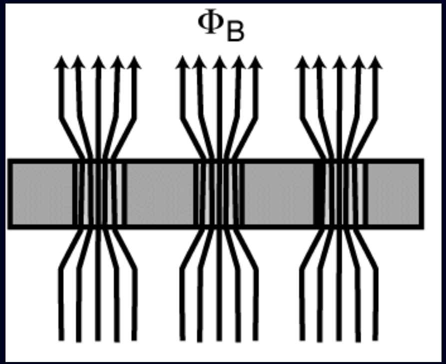


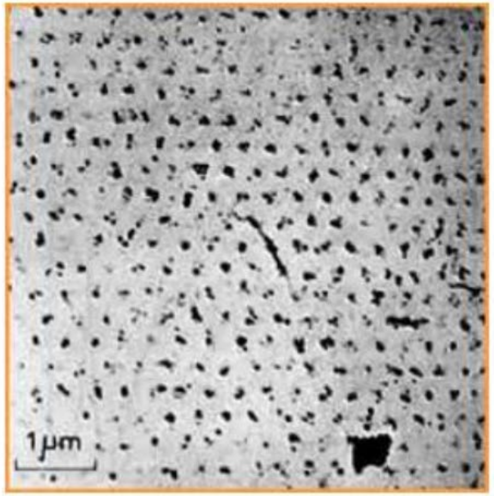




Other ways to image vortices?

Use the magnetic signal





First image of Vortex lattice, 1967

Bitter Decoration

Pb-4at%In rod, 1.1K, 195G

U. Essmann and H. Trauble

Max-Planck Institute, Stuttgart

Physics Letters 24A, 526 (1967)

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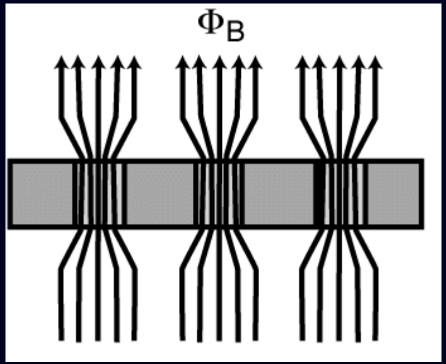
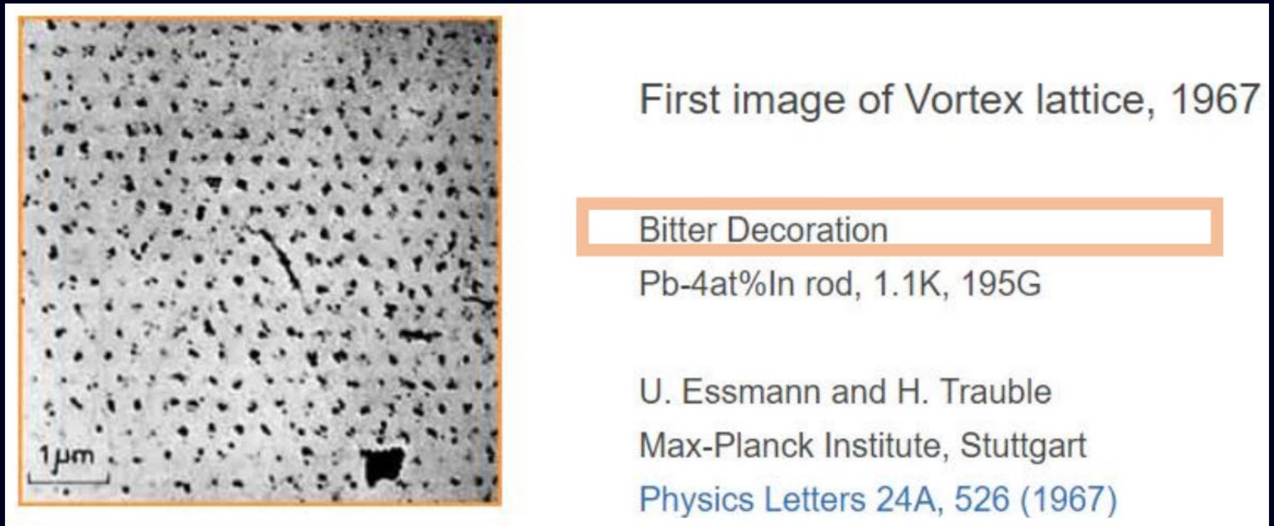
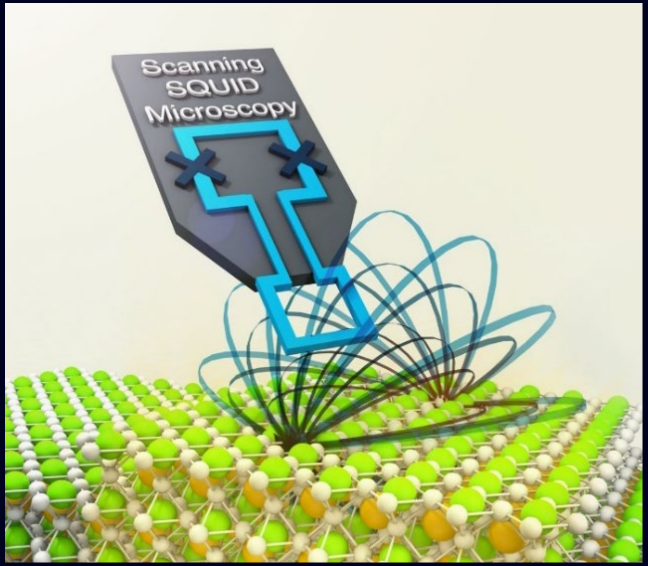
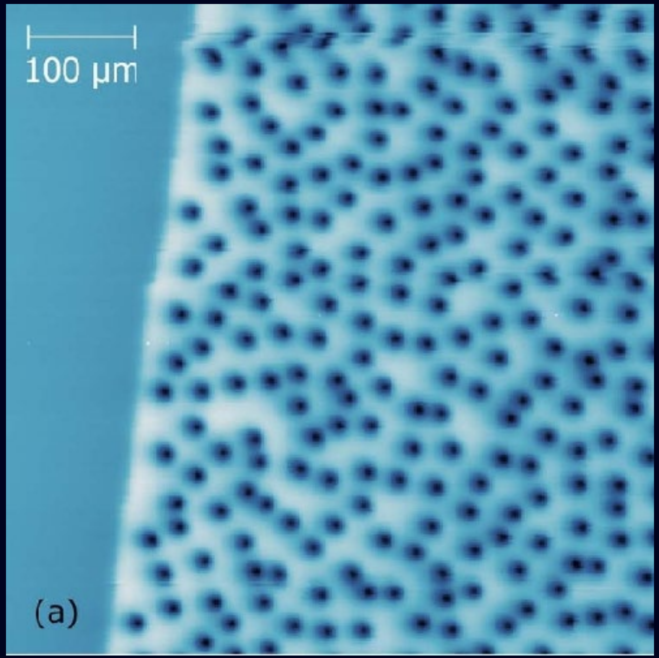




Scanning SQUID



Vortices in YBCO



Wells et al, Scientific Reports (2015)

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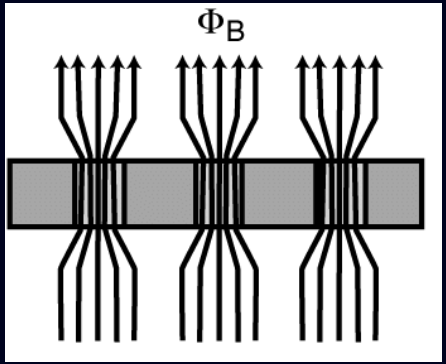
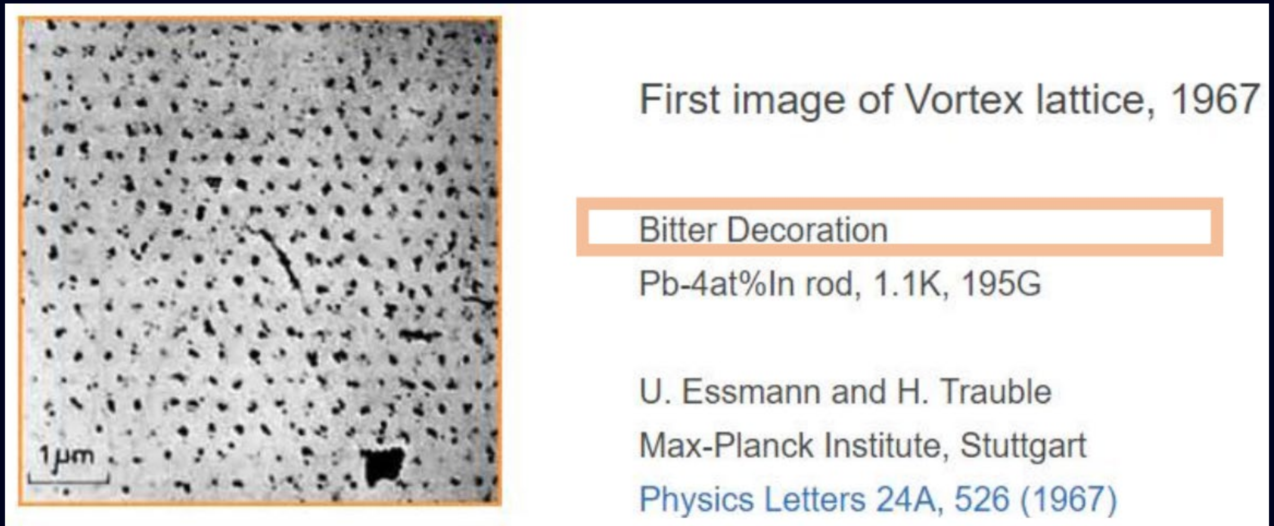
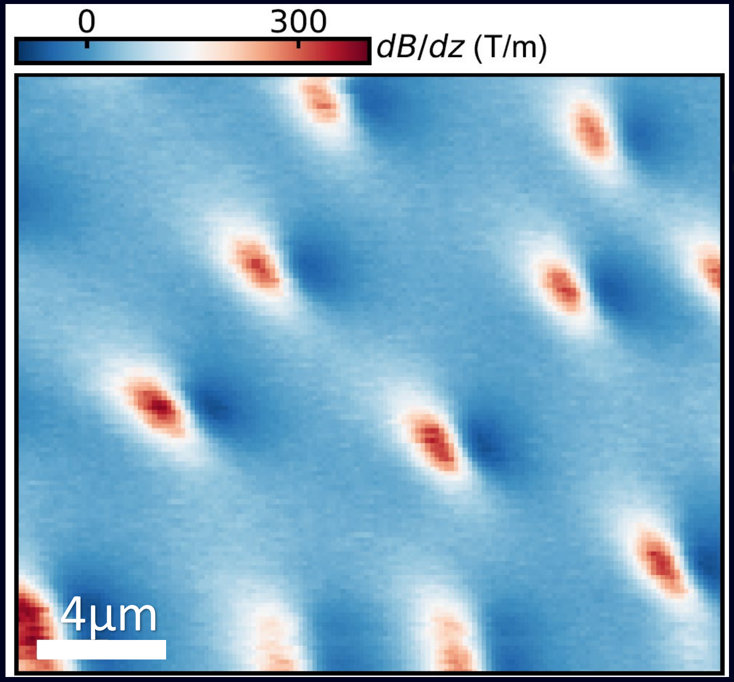




SQUID-on-tip (Lahabi lab)



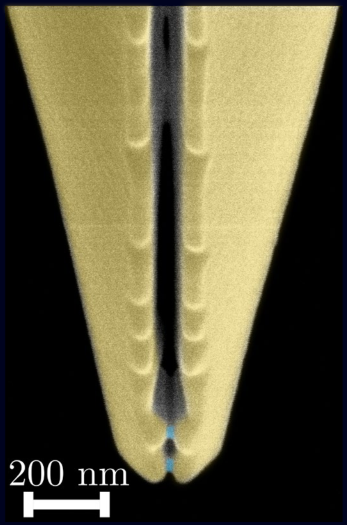
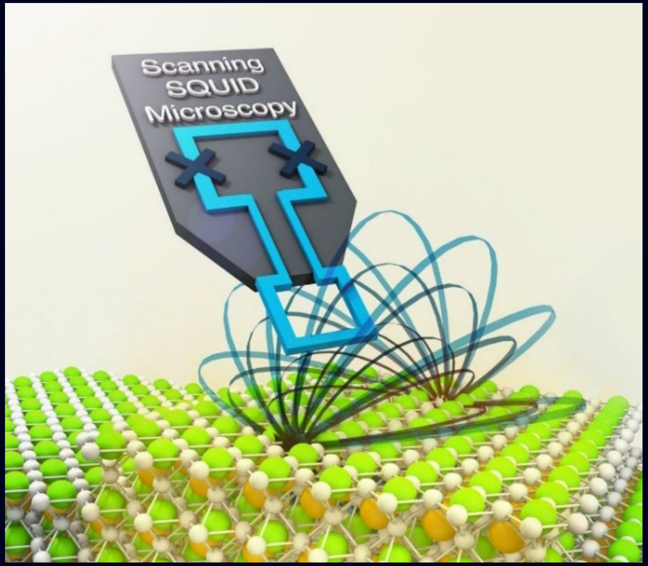


Image taken yesterday!

Scanning SQUID



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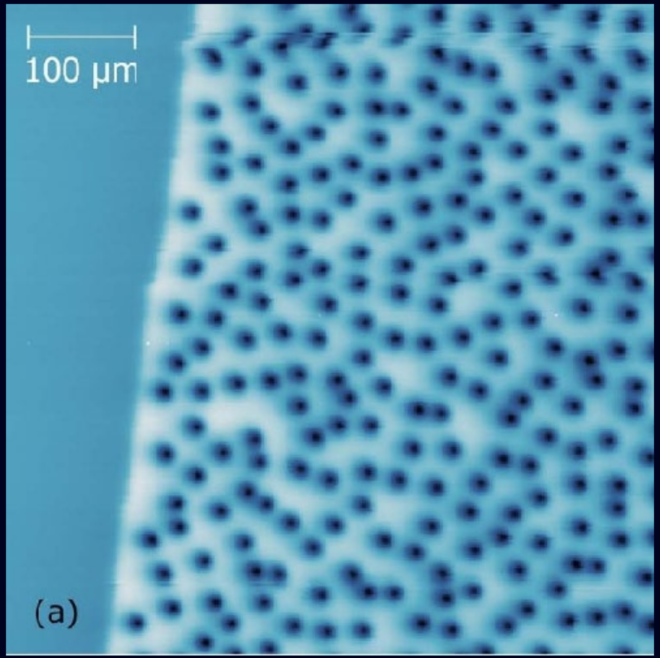
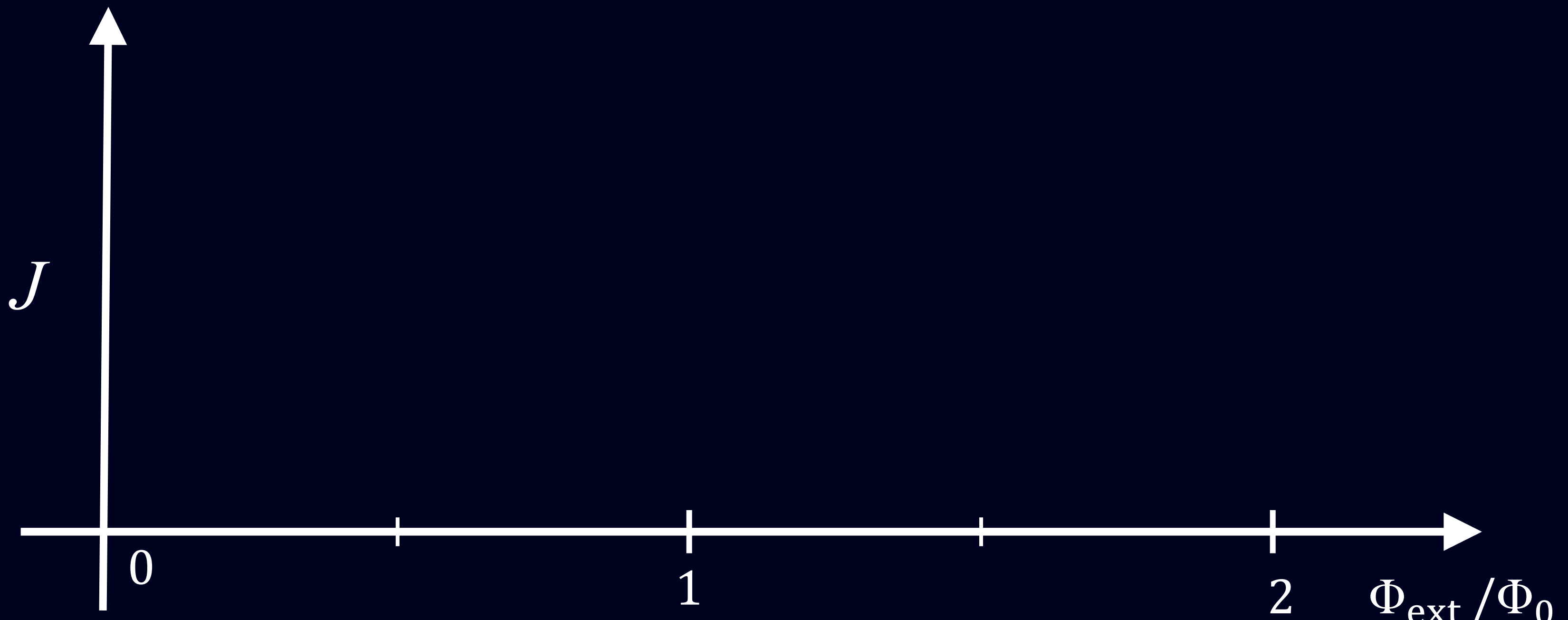
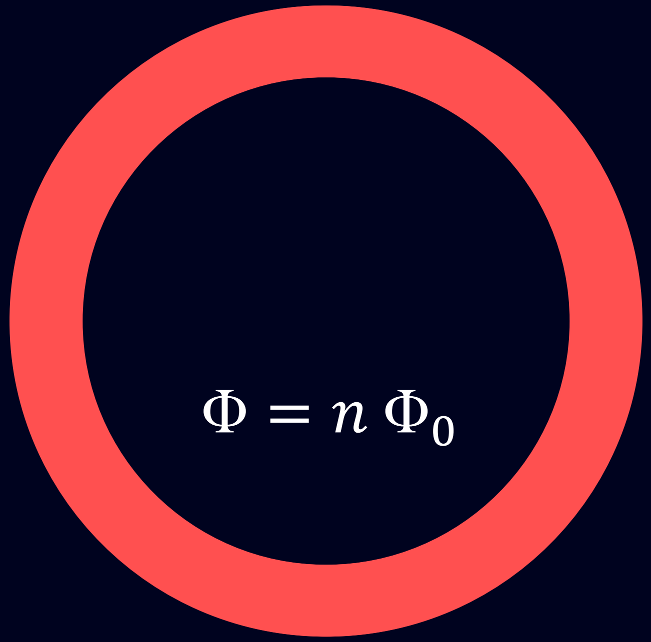


Wells et al, Scientific Reports (2015)

Flux quantization in confined geometries

Thin walls: w < λ (*T*) & w < ξ (*T*)
(no Meissner & uniform |Ψ|)

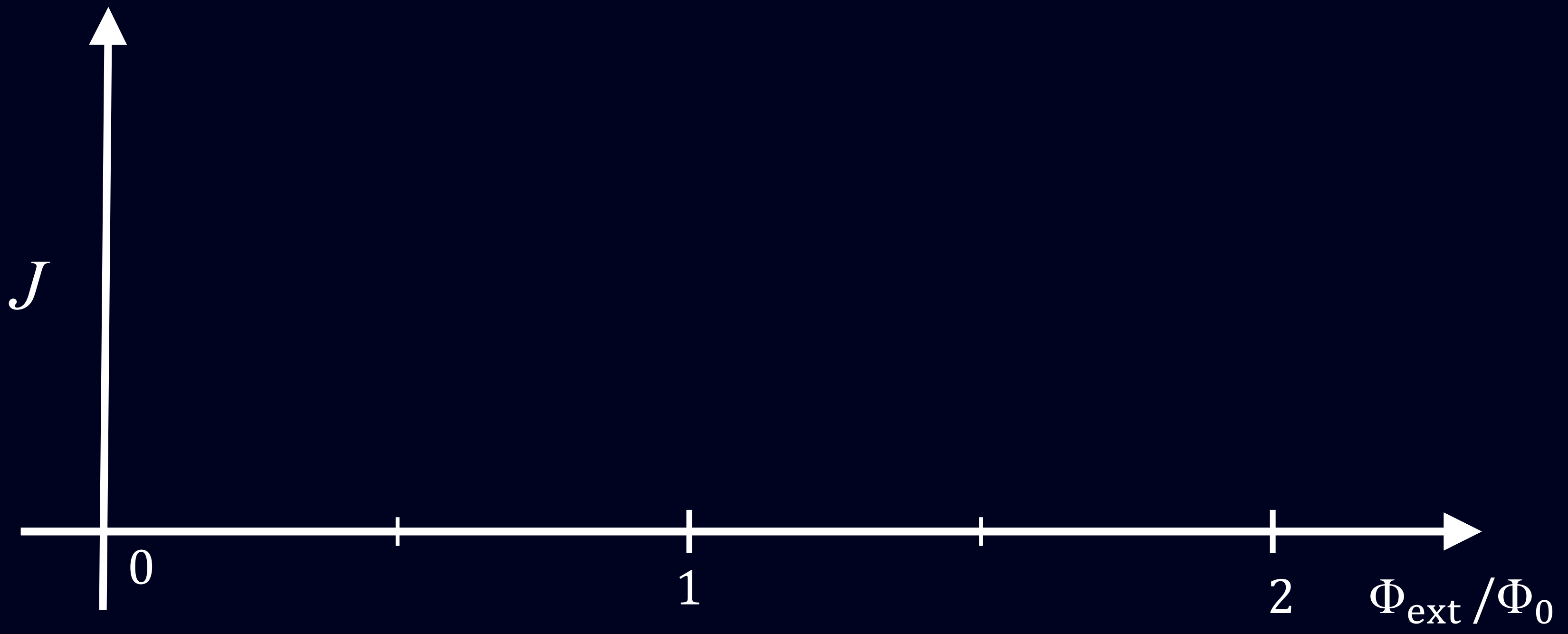




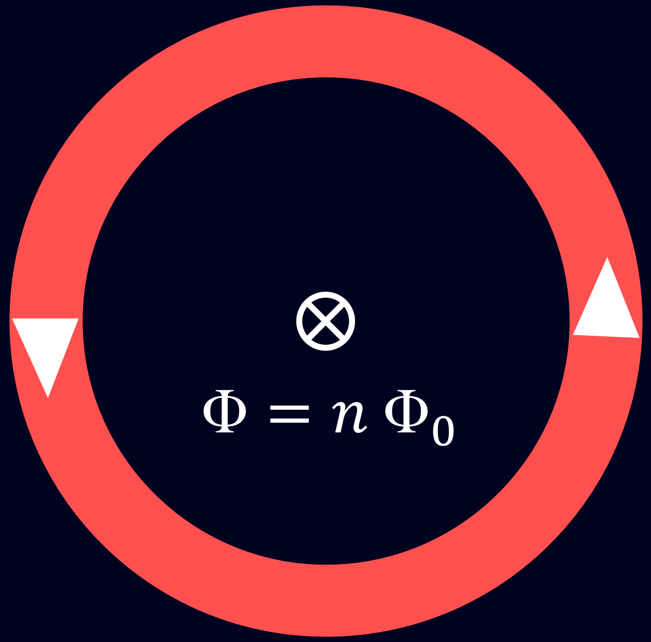
$\Psi = |\Psi| e^{i\varphi}$

φ needs to wind by n × 2π around the loop ⟶ $\Phi = n\,\Phi_0$

Thin walls: w < λ (*T*) & w < ξ (*T*)
(no Meissner & uniform $|\Psi|$)



$\Phi_{ext} \neq n\Phi_0, J \neq 0$



$\Psi = |\Psi| e^{i\varphi}$

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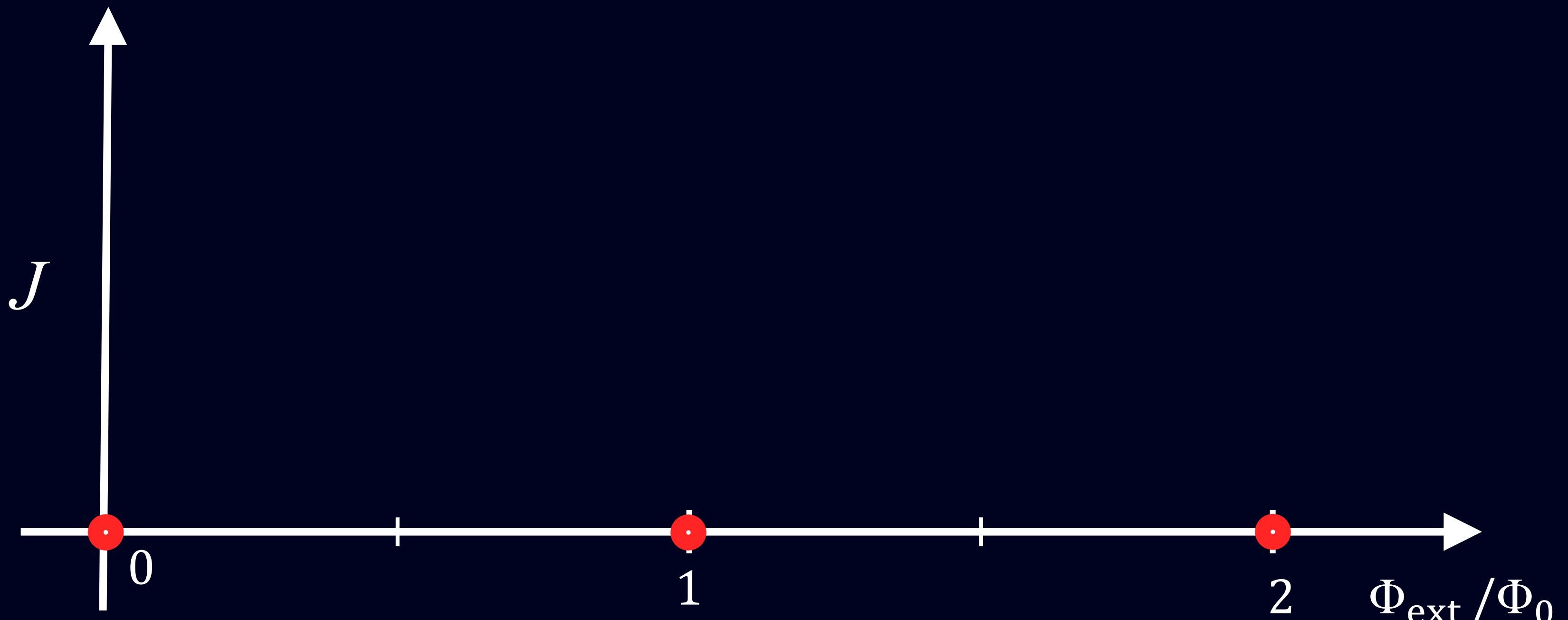
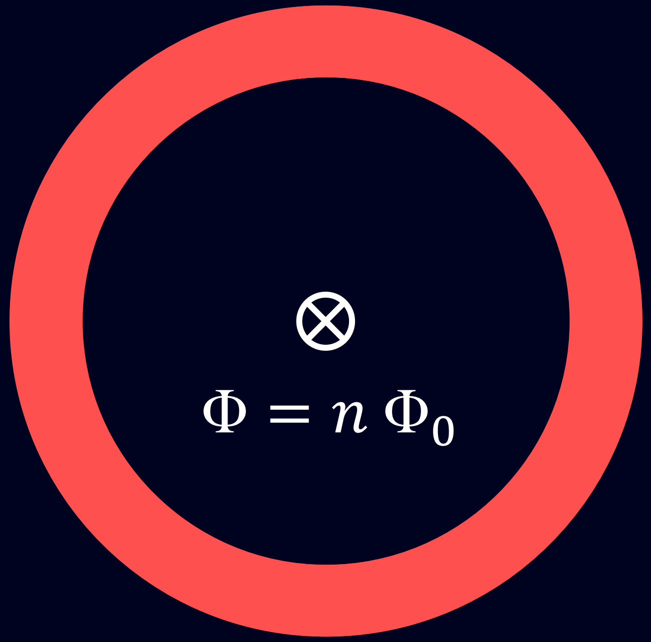
If $\Phi_{ext} \neq n\Phi_0$ → a circulating current J compensates for the phase offset

Thin walls: w < λ (T) & w < ξ (T)
(no Meissner & uniform $|\Psi|$)



$\Phi_{ext} = 0, \Phi_0, 2\Phi_0 \ldots \rightarrow J = 0$



$\Psi = |\Psi| e^{i\varphi}$

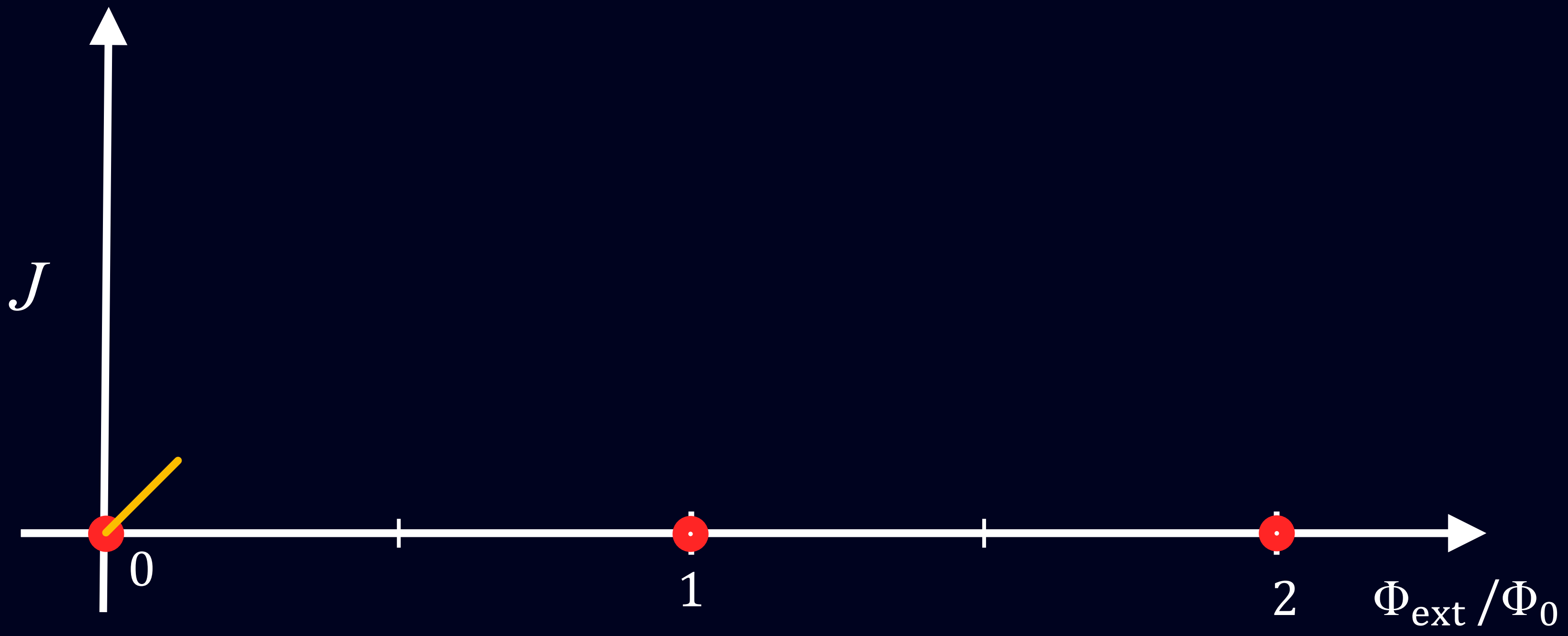
φ needs to wind by n × 2π around the loop $\longrightarrow$ $\Phi = n\,\Phi_0$

If $\Phi_{ext} \neq n\Phi_0 \rightarrow$ a circulating current J compensates for the phase offset

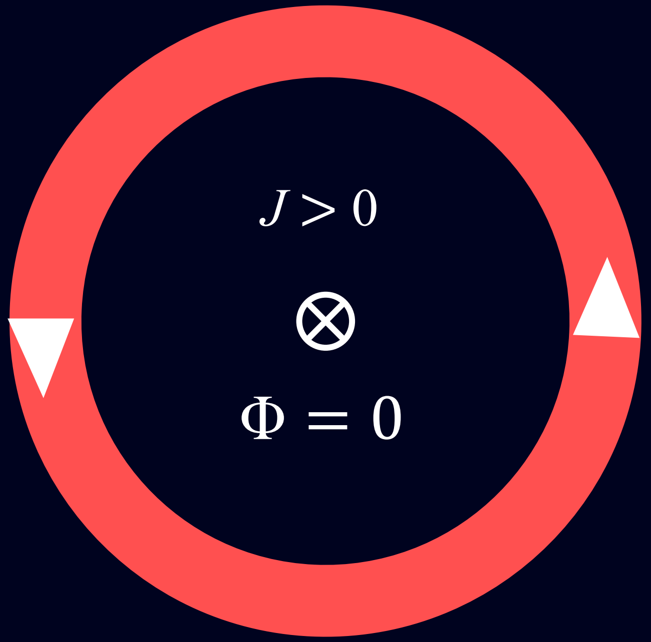
However, supercurrents still cost kinetic energy & should be minimized $\rightarrow \Phi_{ext} = n\Phi_0, J = 0$

Let’s ramp up the external field:

Remember $\Phi_{ext} = \mu_0 H_{ext} \times$ (Ring area)

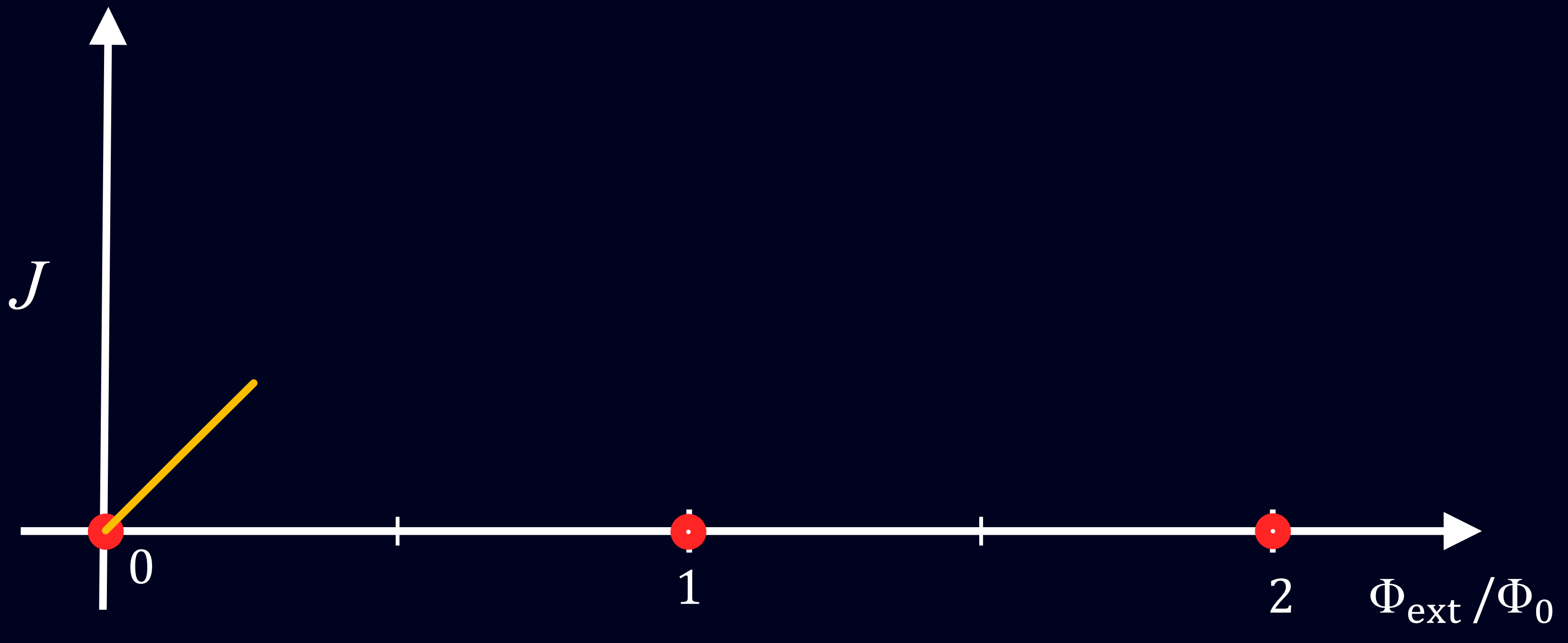


$0 < \Phi_{ext} < \Phi_0/2$

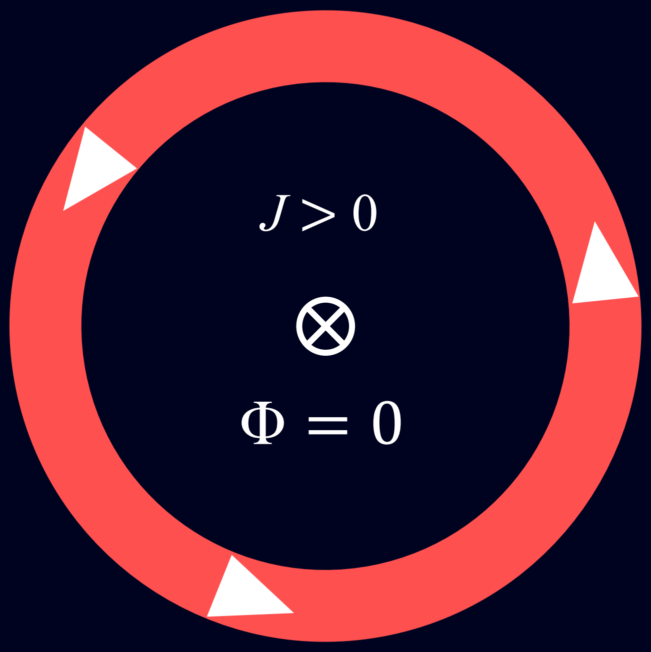


A supercurrent starts to circulate in the ring

Let’s ramp up the external field: A bit more

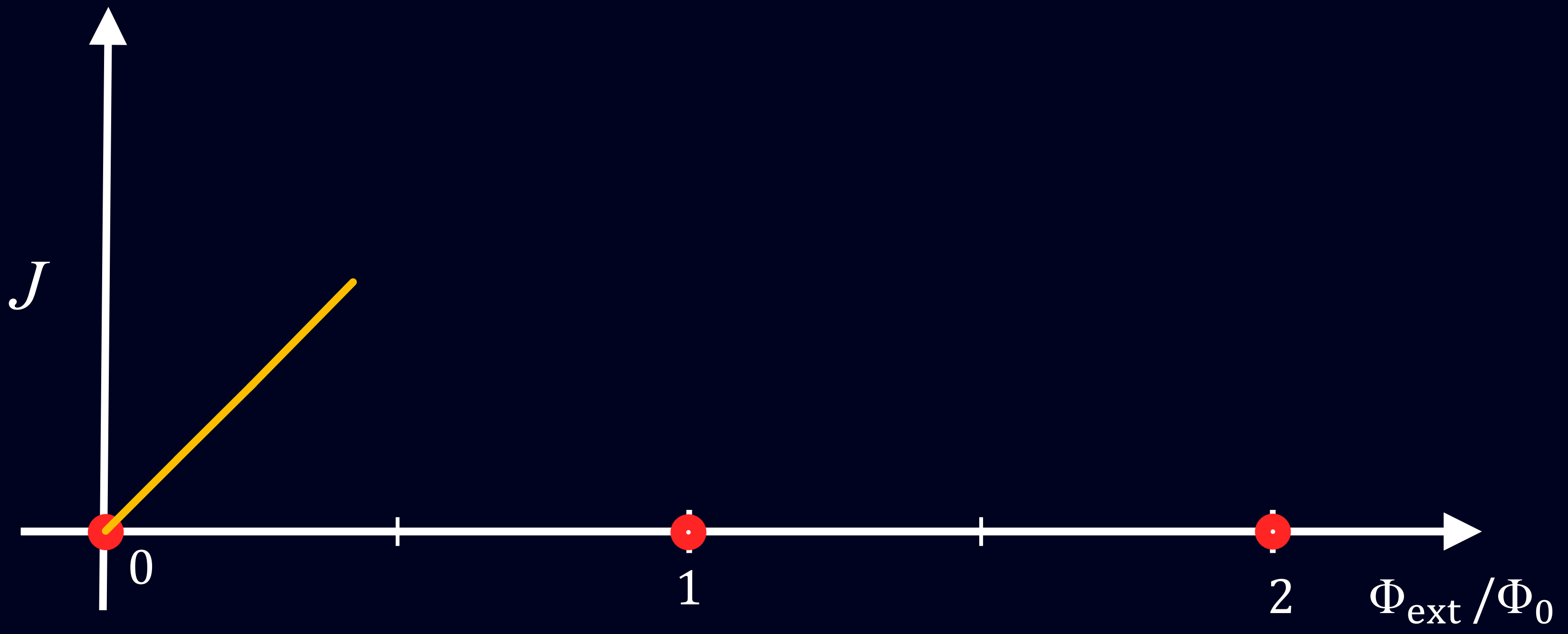


$0 < \Phi_{ext} < \Phi_0/2$

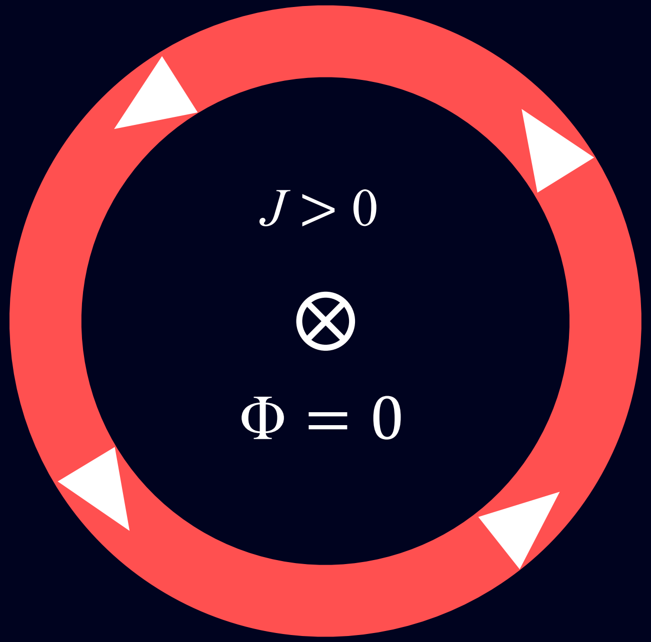


The superconductor needs to work harder to compensate for the phase offset
→ More supercurrent

Let’s ramp up the external field: Keep going

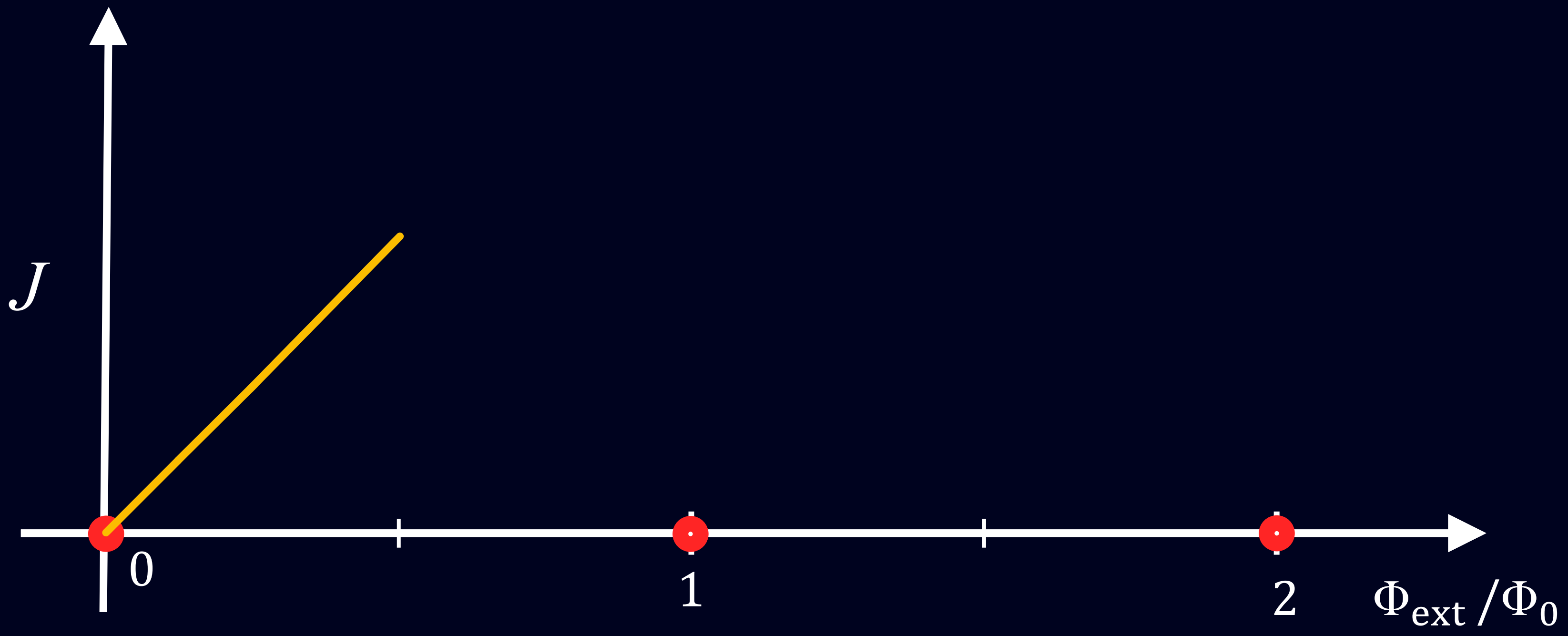


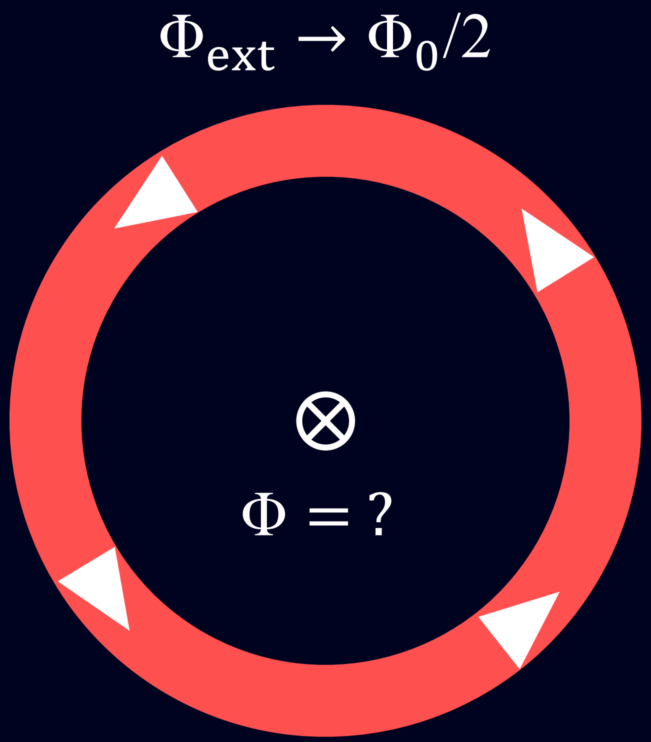
$0 < \Phi_{\text{ext}} < \Phi_0/2$

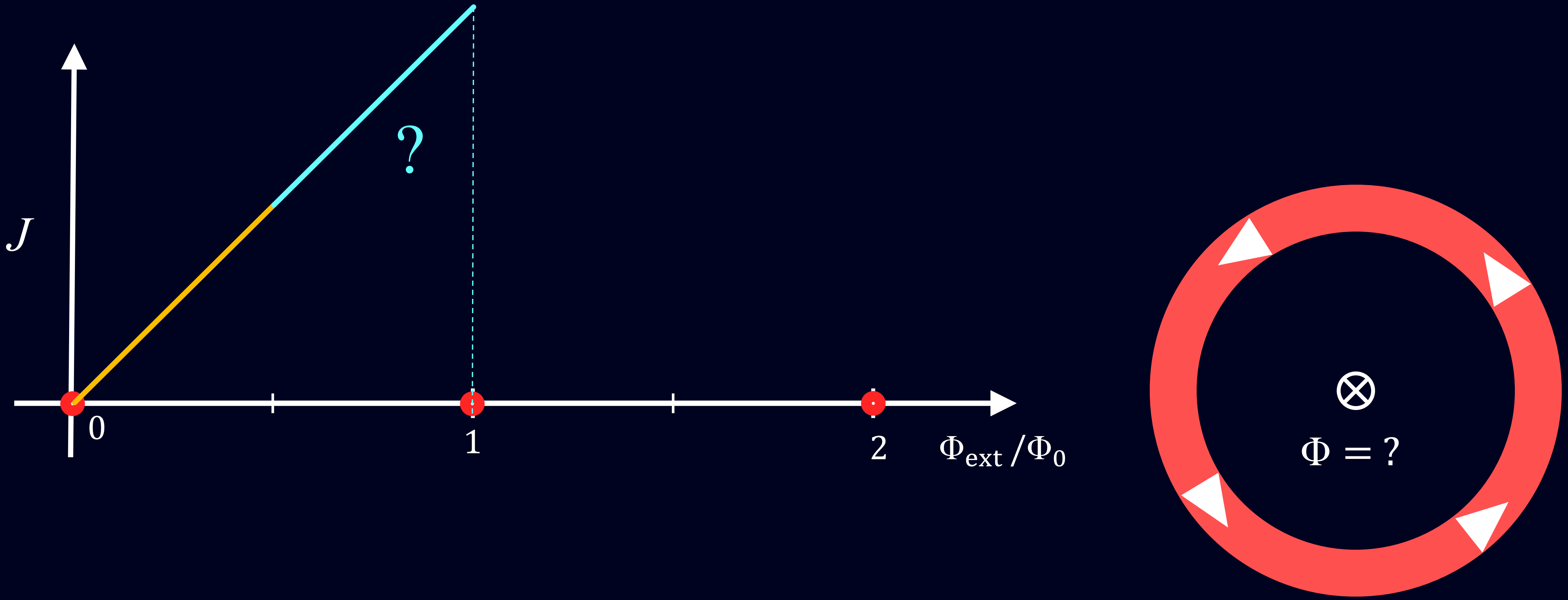


Supercurrent increases its veolcity to keep up

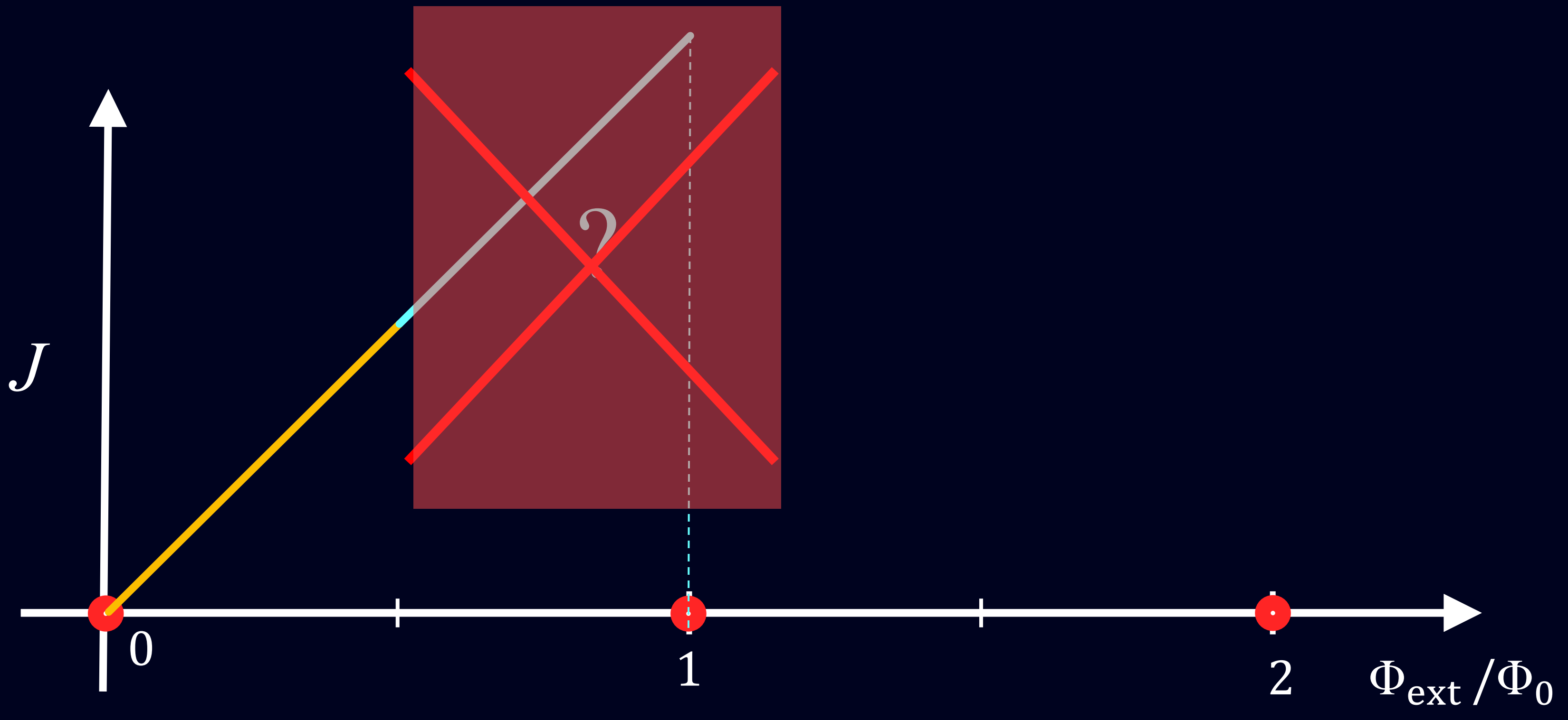
But what happens as Φ_{ext} goes above $\Phi_0/2$?





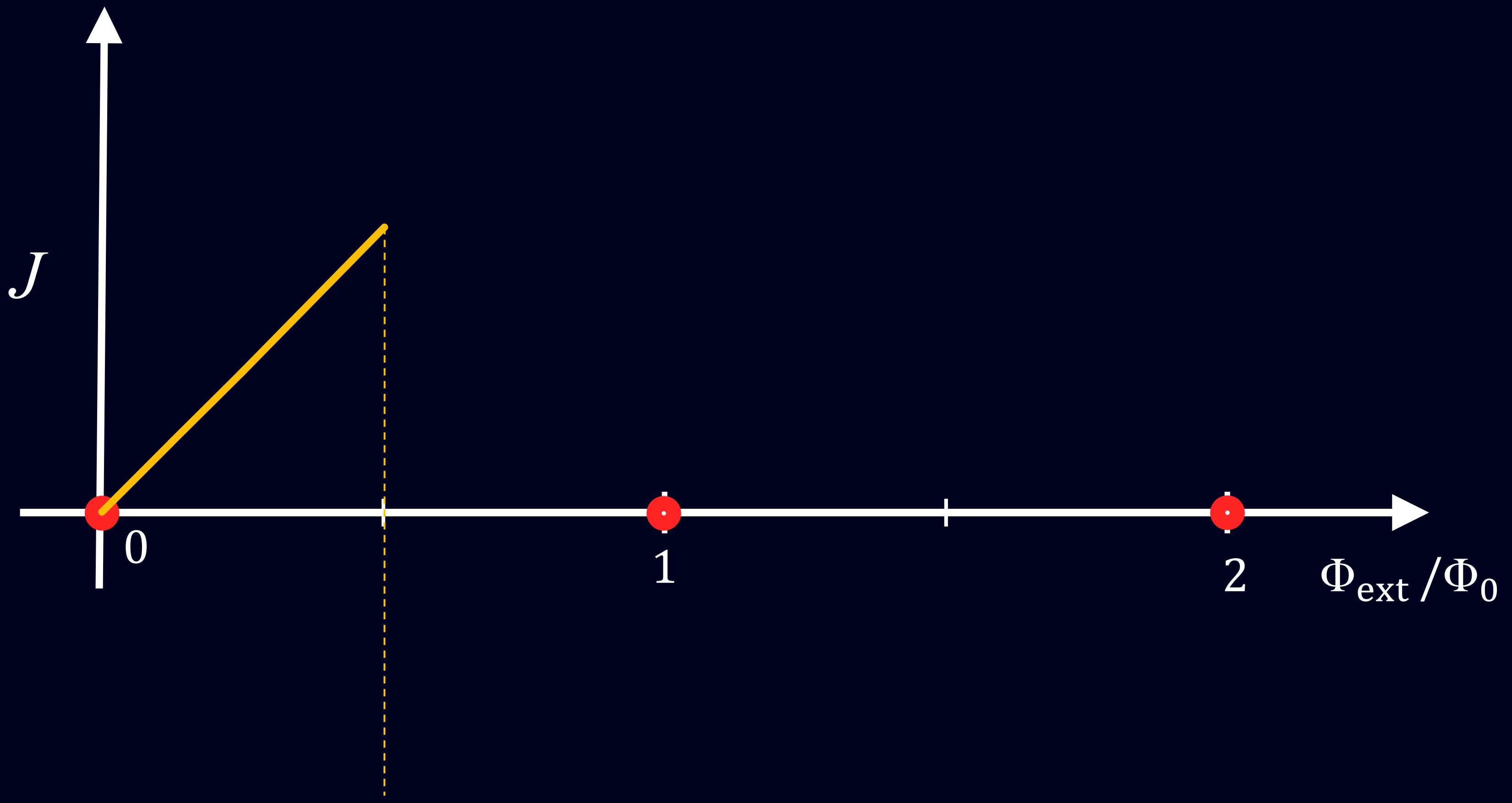


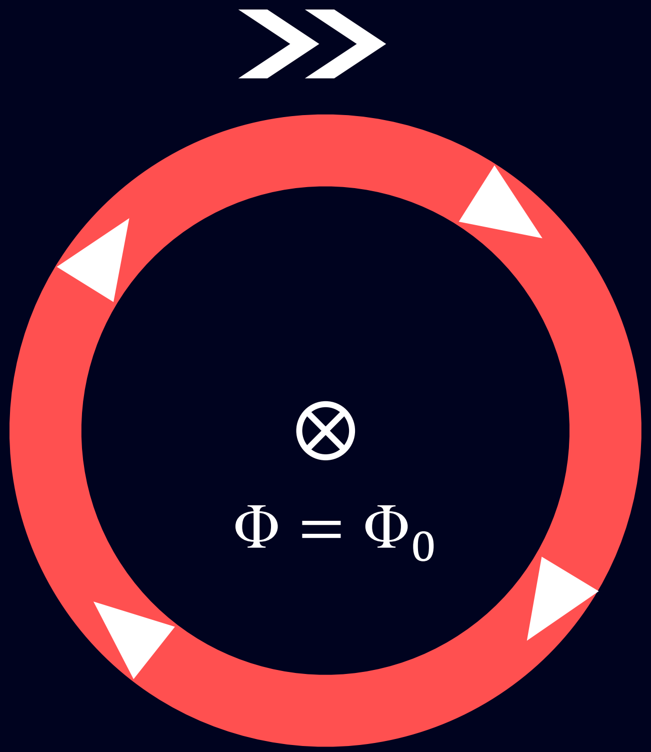
Should the suppercurrent keep increasing its velocity to cencel out the external flux (continue as $\Phi = 0$) until it reaches Φ_0?



Should the suppercurrent keep increasing its velocity to cencel out the external flux (continue as $\Phi = 0$) until it reaches Φ_0?

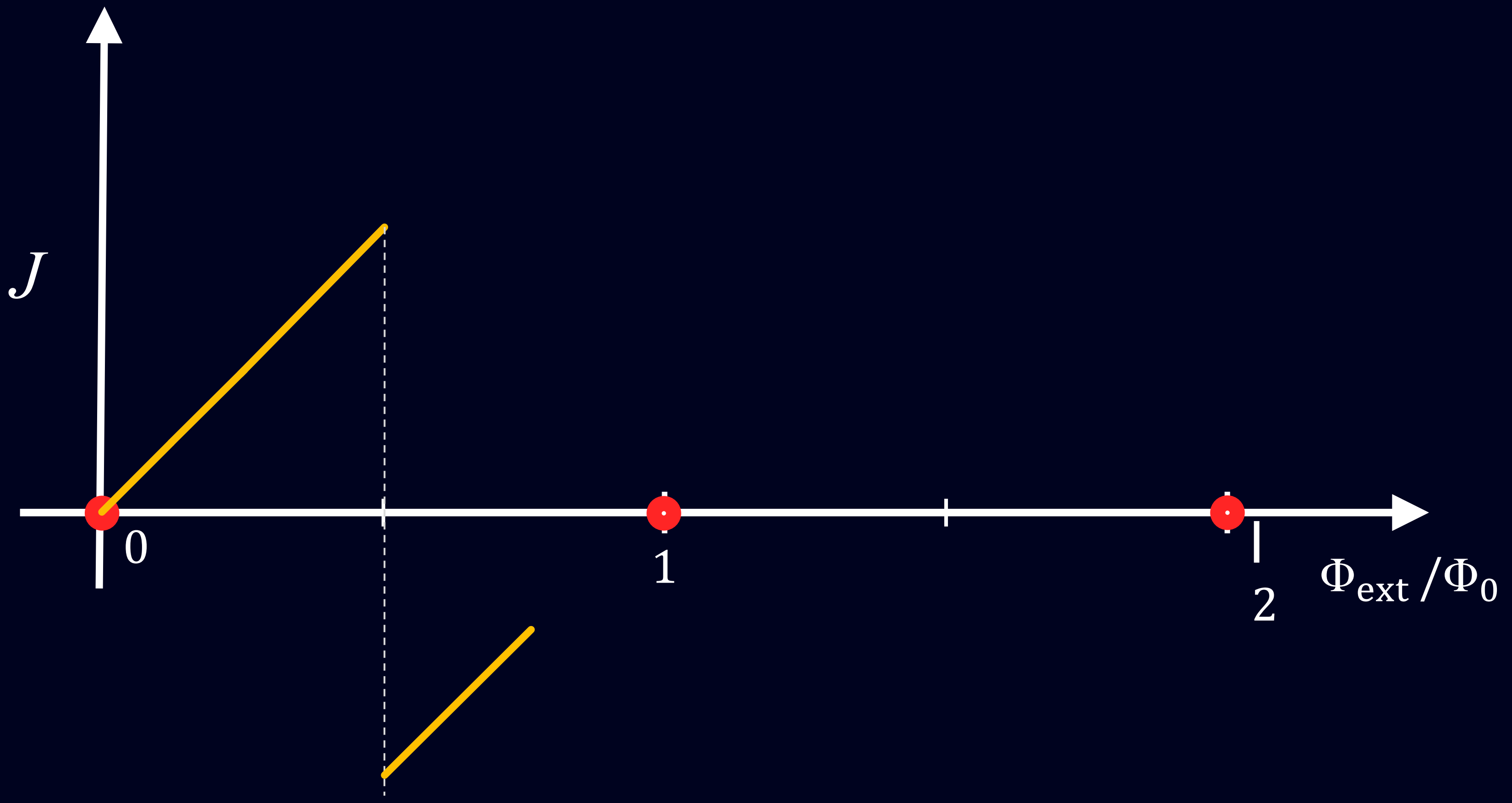
No, there’s a better way!

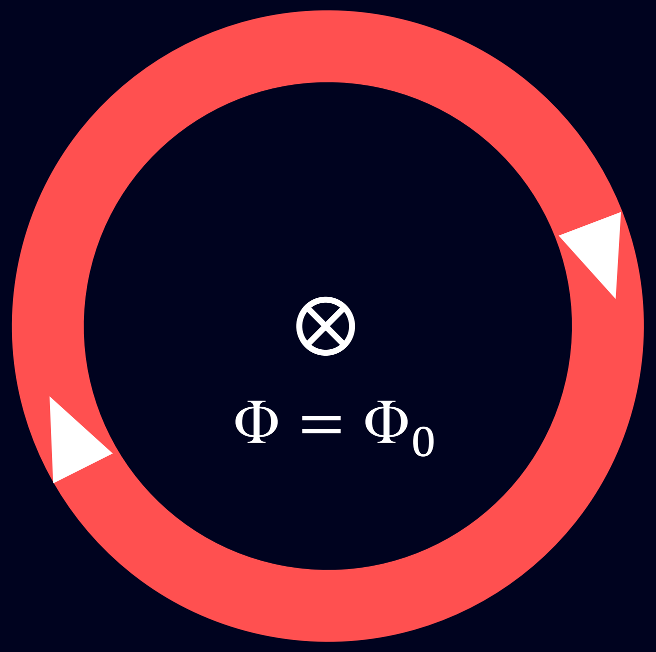




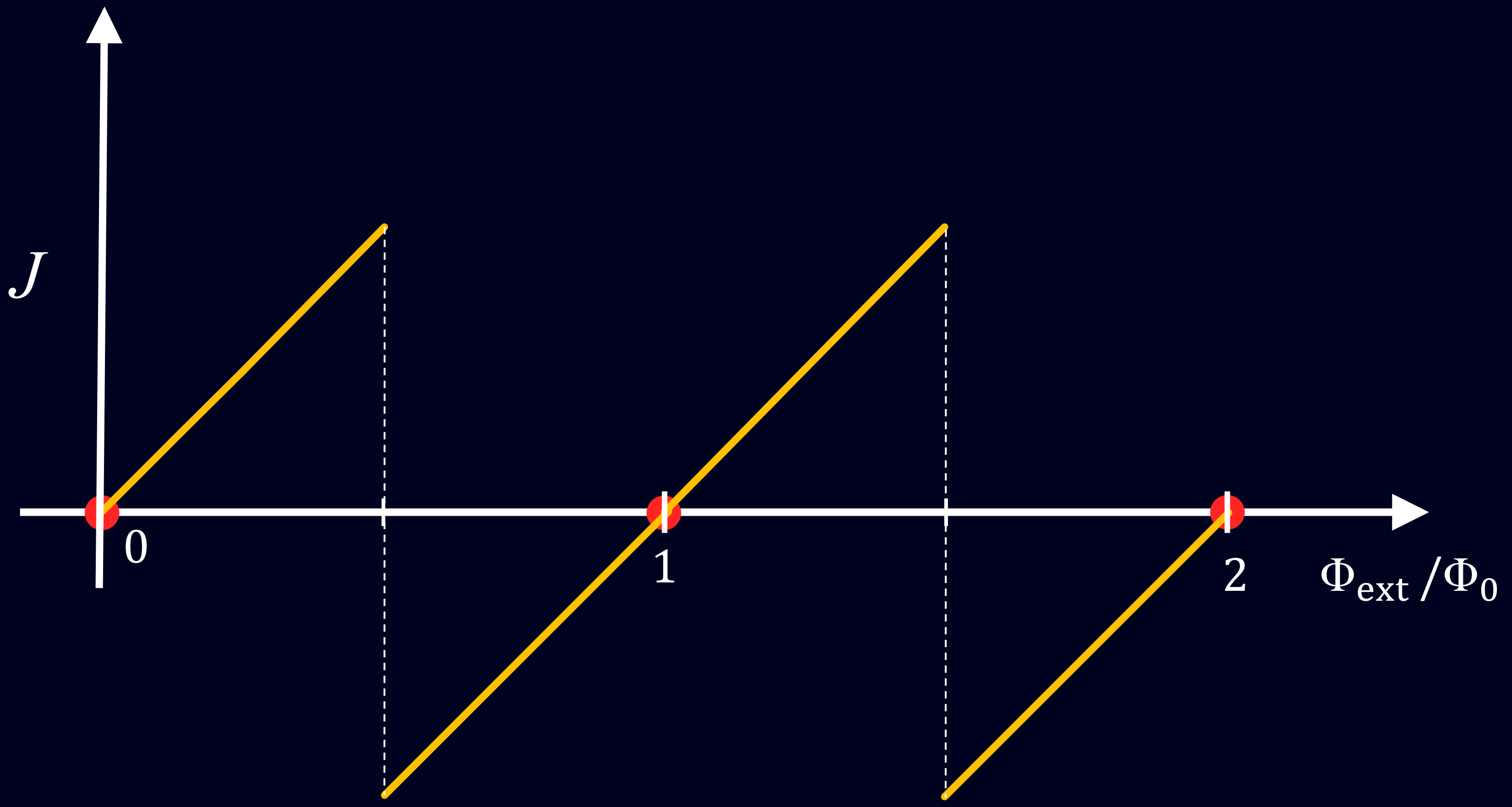
Superconductor saves its energy by switching the direction of J above $\Phi_0/2$, so that $1\Phi_0$ can enter the loop, even though $\Phi_{ext} < \Phi_0$

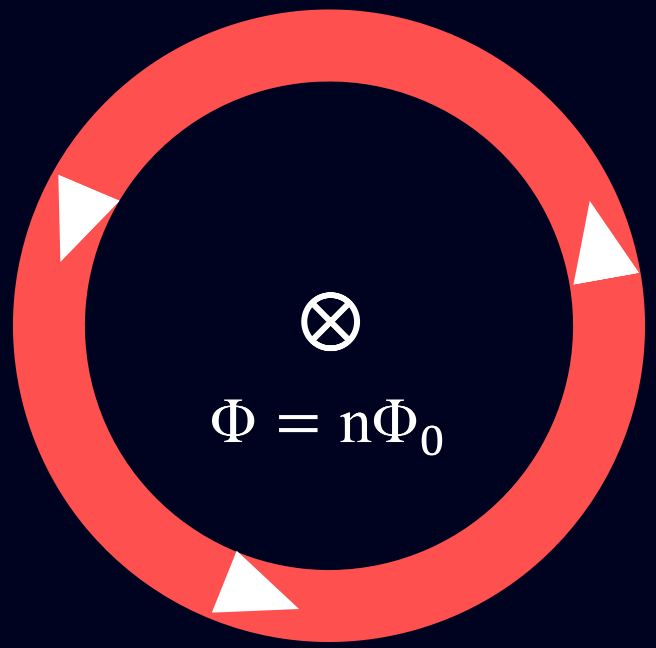
This means the supercurrent starts to amplify the flux (instead of cancelling it)!





The supercurrent winds down as we increase the field above $\Phi_0/2$ and J stops when $\Phi_{ext} = \Phi_0$





This cycle repeats every Φ_0

J

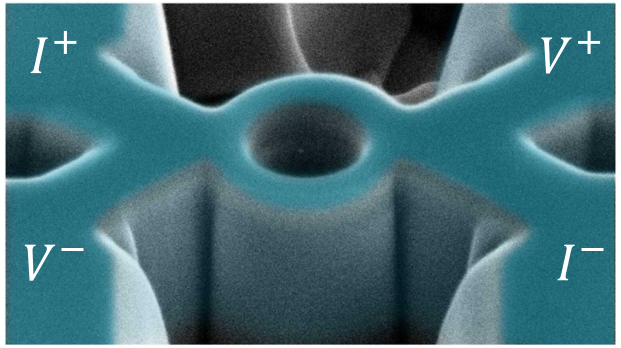
Moral of the story:

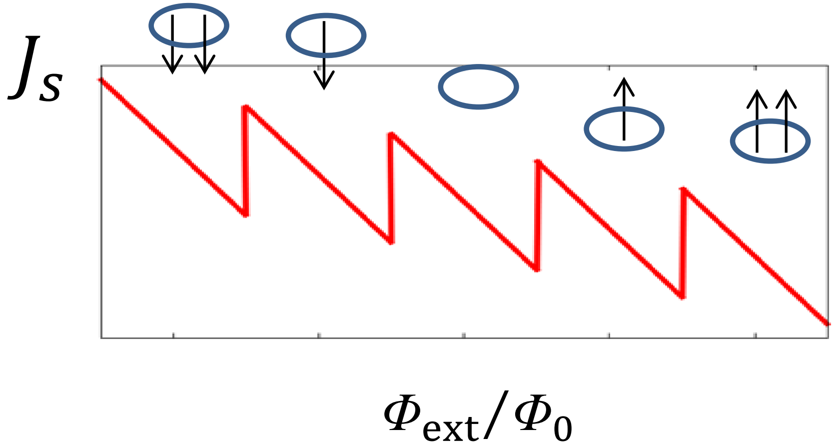
1. Superconductors don't just screen magnetic fields, they can also amplify it! All they care about is that their wavefunction remains single-valued, i.e., that their phase can wind continuously by integer multiples of 2π.

2. Unlike in normal metals, where a current can be generated by a changing magnetic field (J ∝ dB/dt). In SCs, the supercurrent scales with the value of magnetic flux (not its rate, $d\Phi/dt$) and its relation with Φ_0.

This cycle repeats every Φ_0

Little-Parks Experiment





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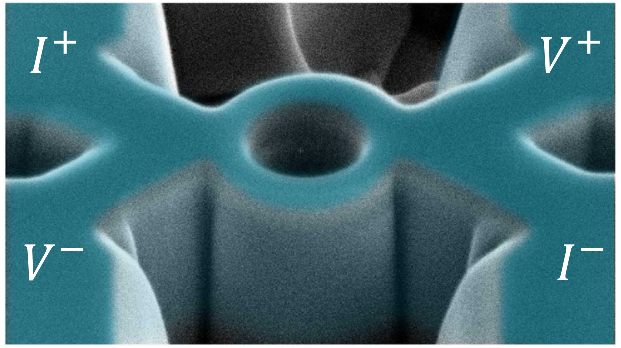

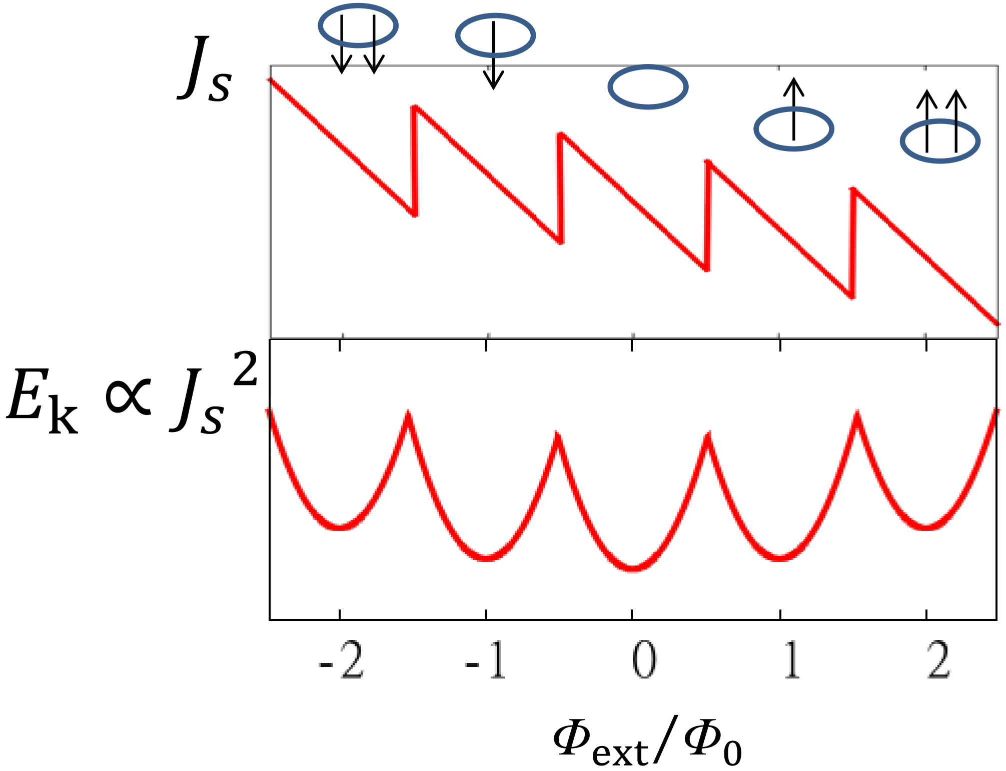

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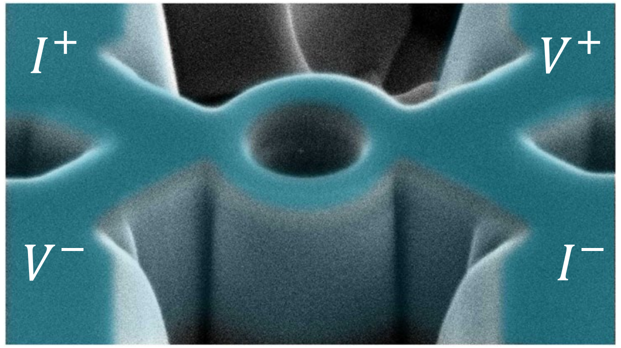

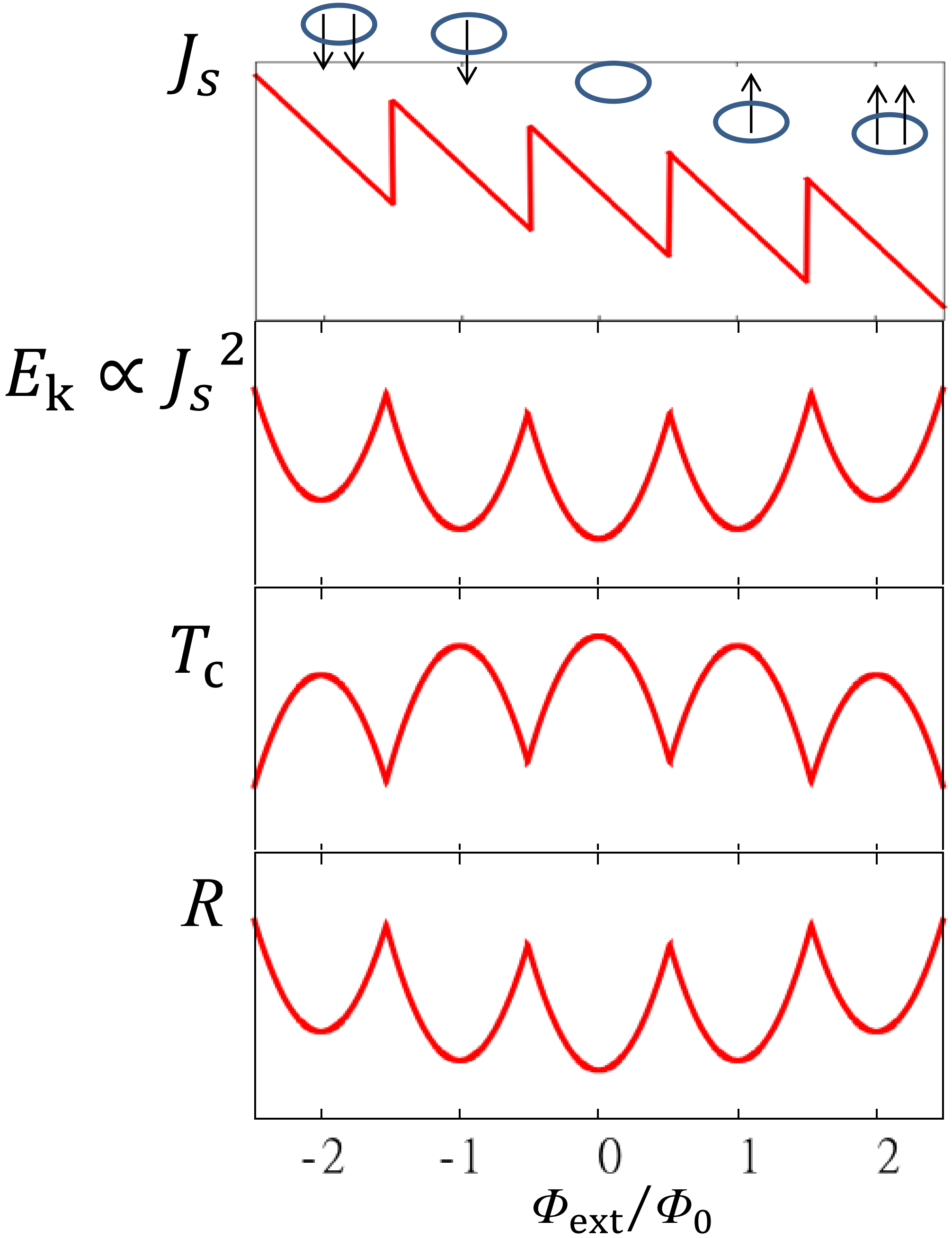

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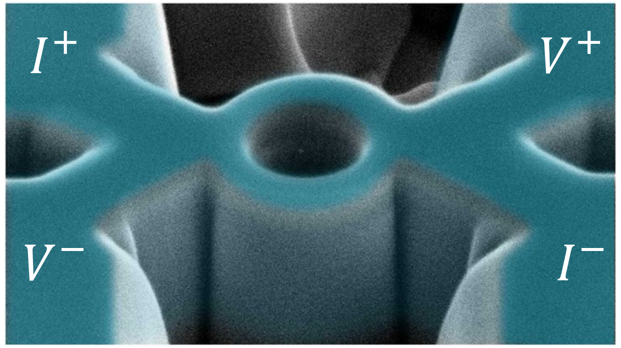

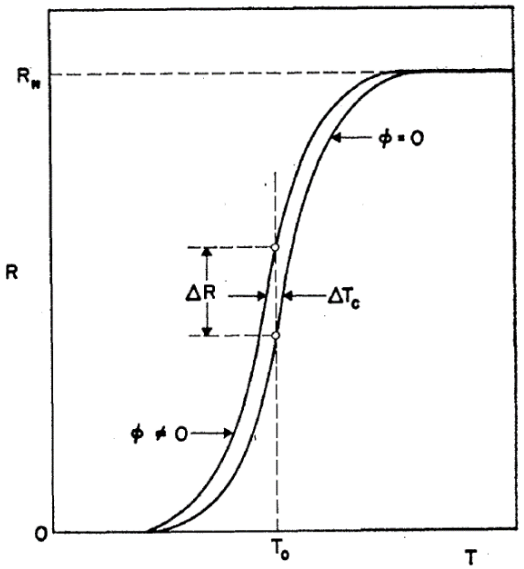

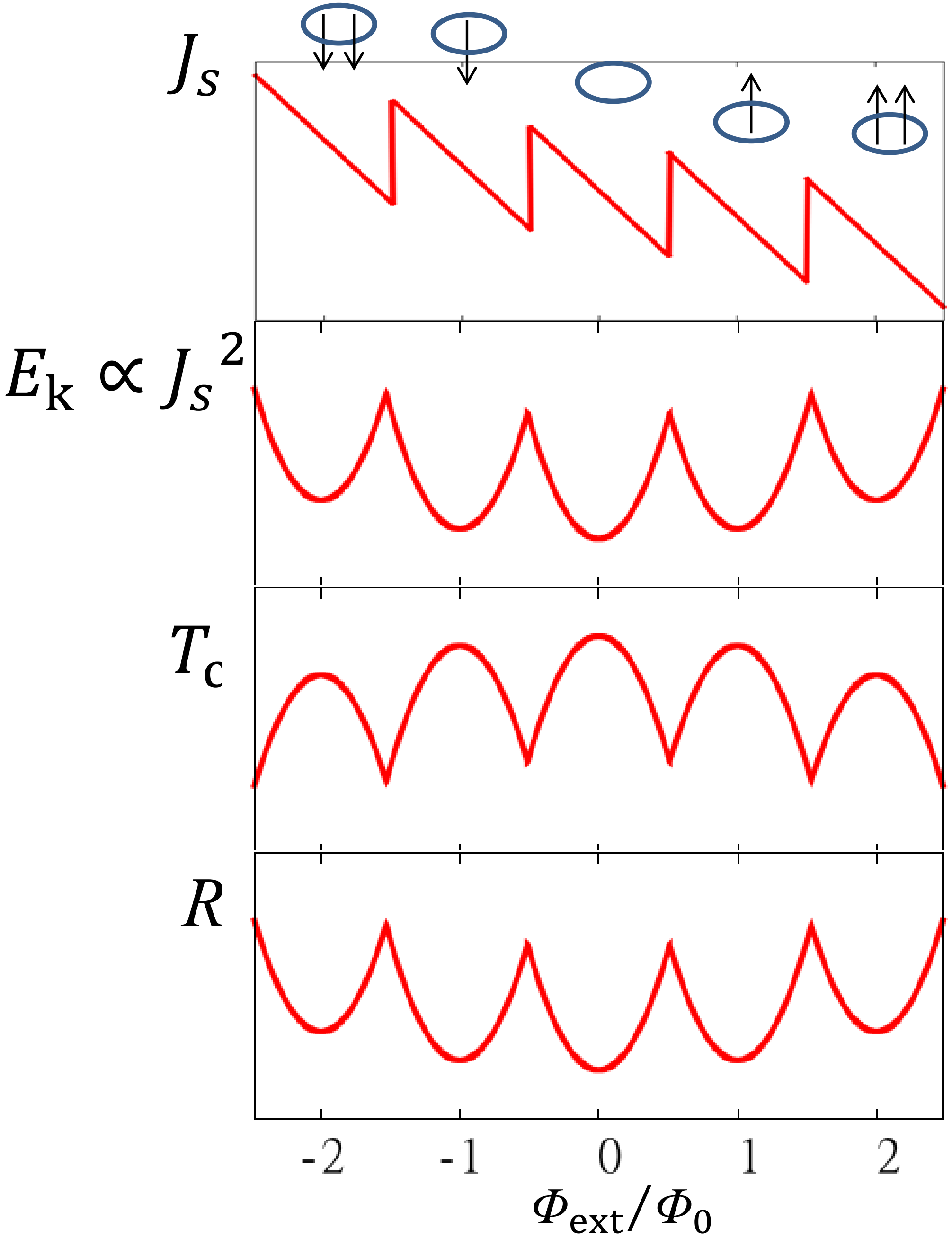

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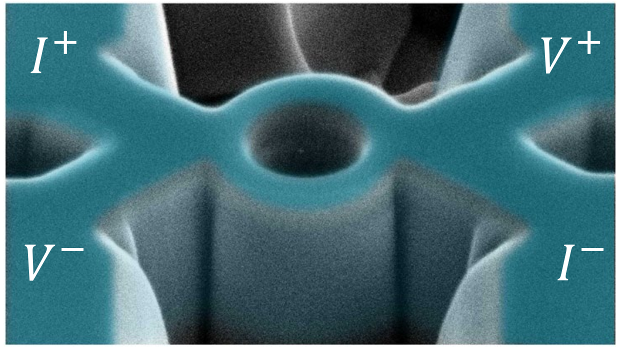
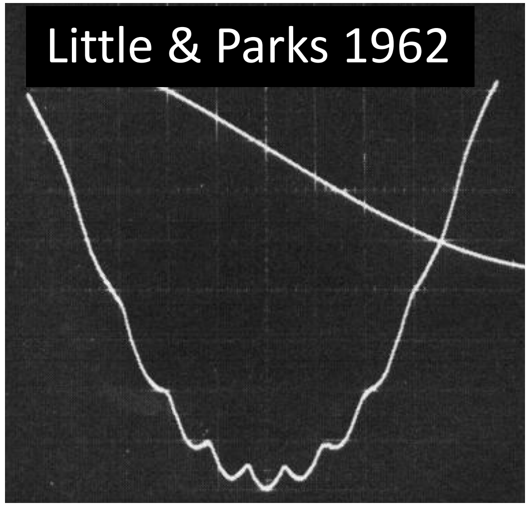
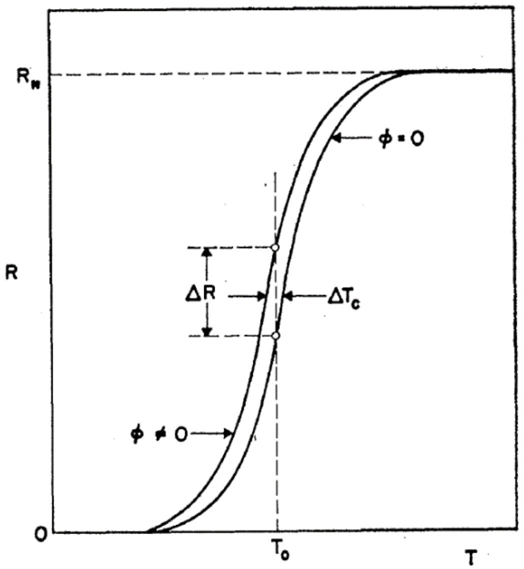
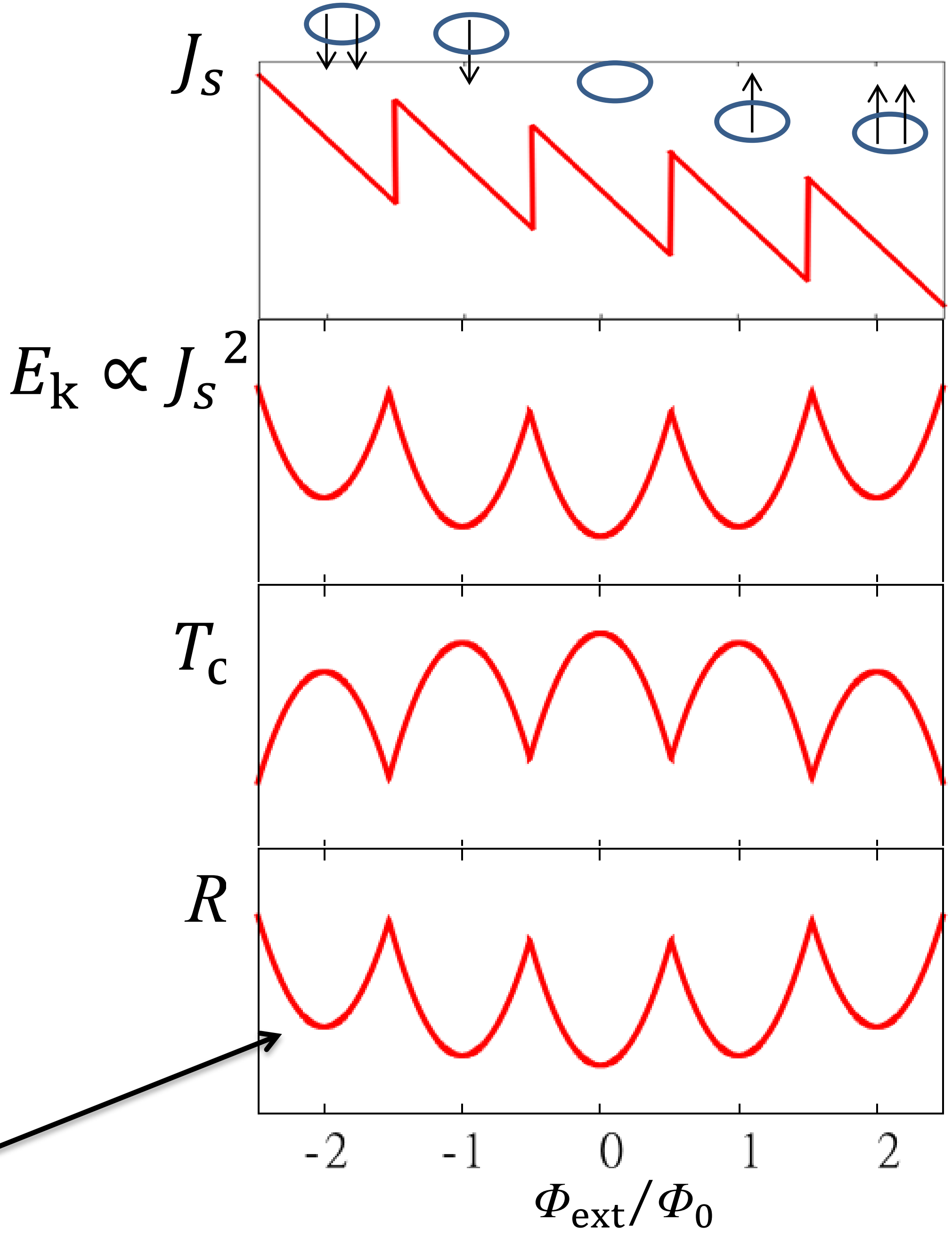




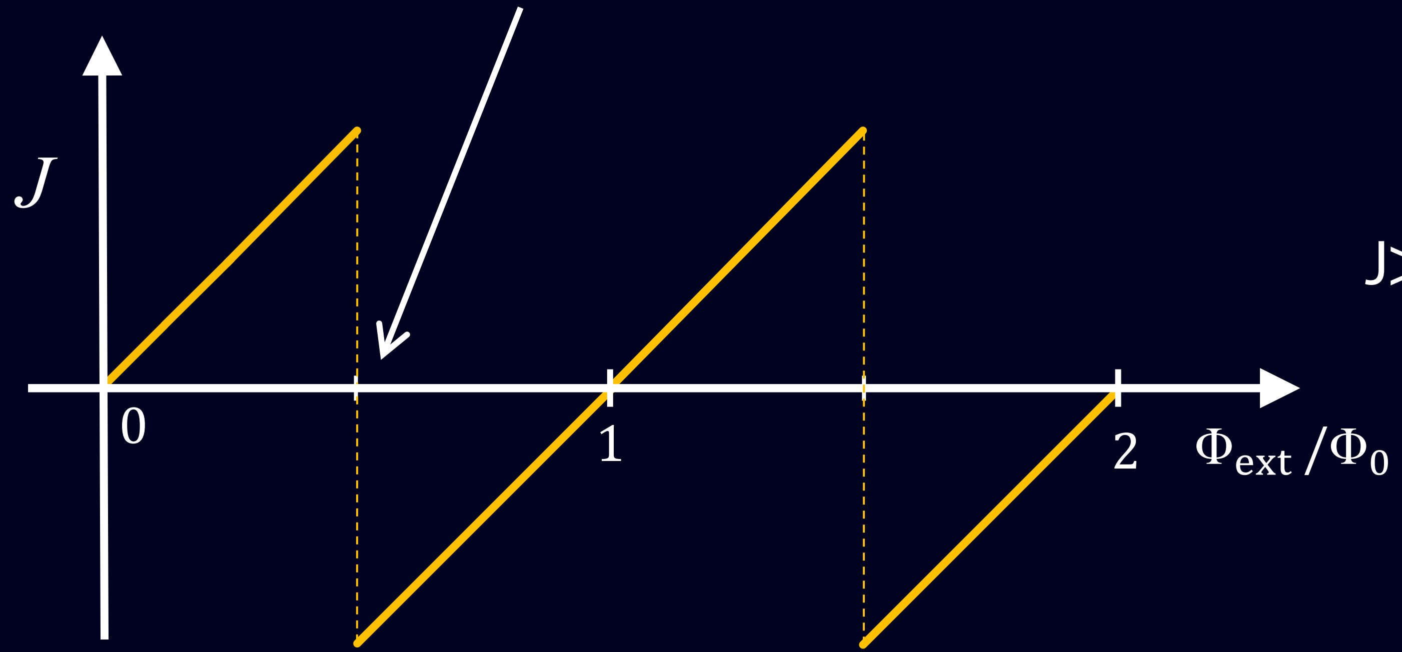
Period = $\frac{h}{\mathbf{2e}}$



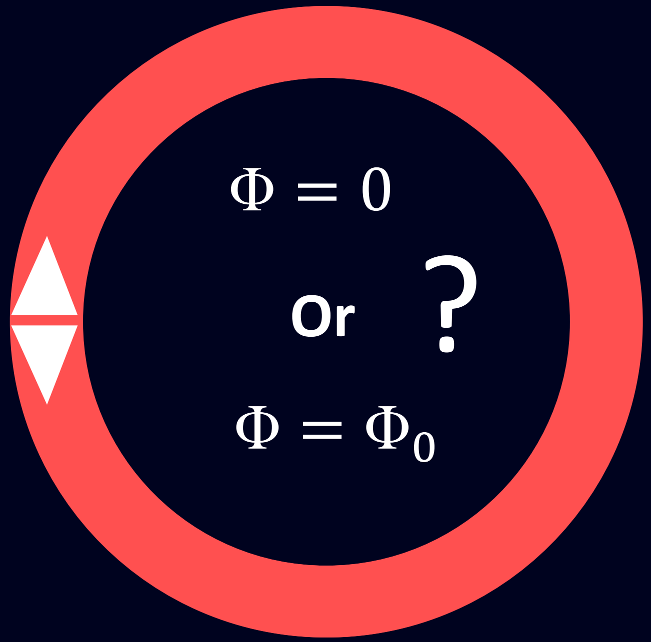
Why the parabolic background?
(see later & Moshchalkov)



What's happening at $\Phi_{ext} = \Phi_0/2$ exactly?



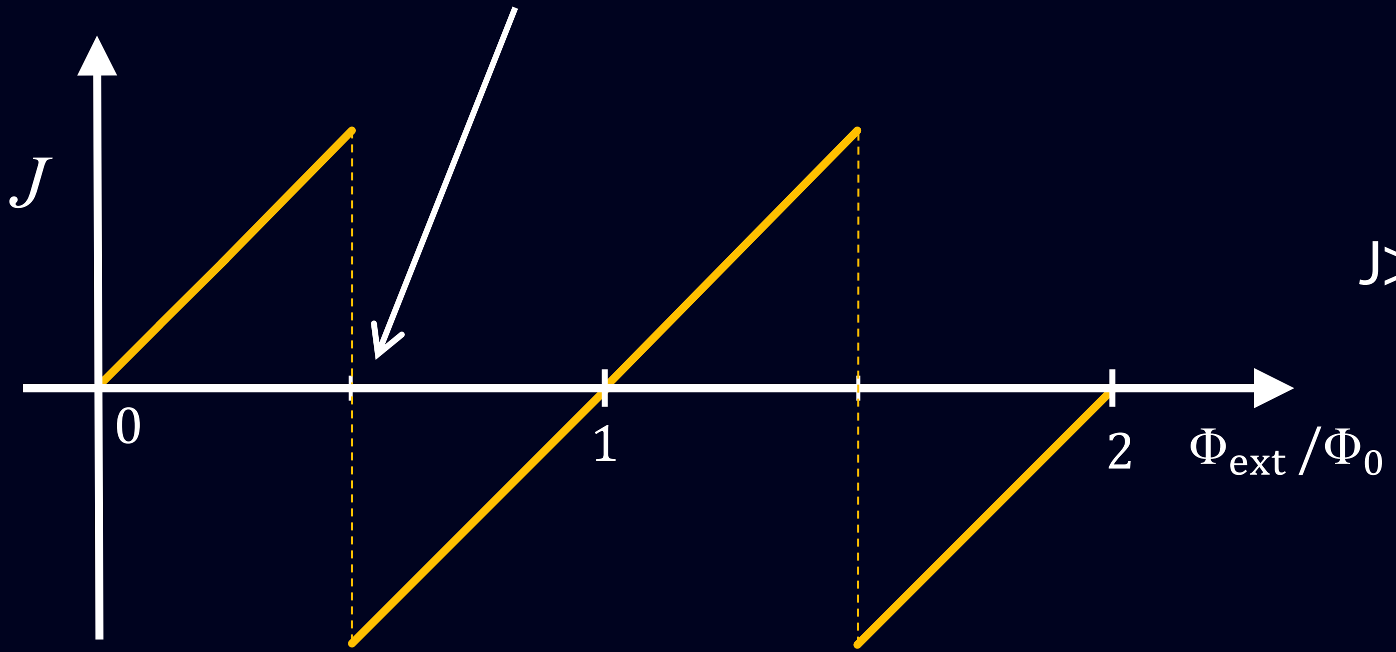
J>0 or J> 0 ??



Which way does the supercurrent circulate if Φ_{ext} is exactly $\Phi_0/2$

What's the flux inside the ring? 0 or Φ_0?

What's happening at $\Phi_{\mathrm{ext}} = \Phi_0/2$ exactly?



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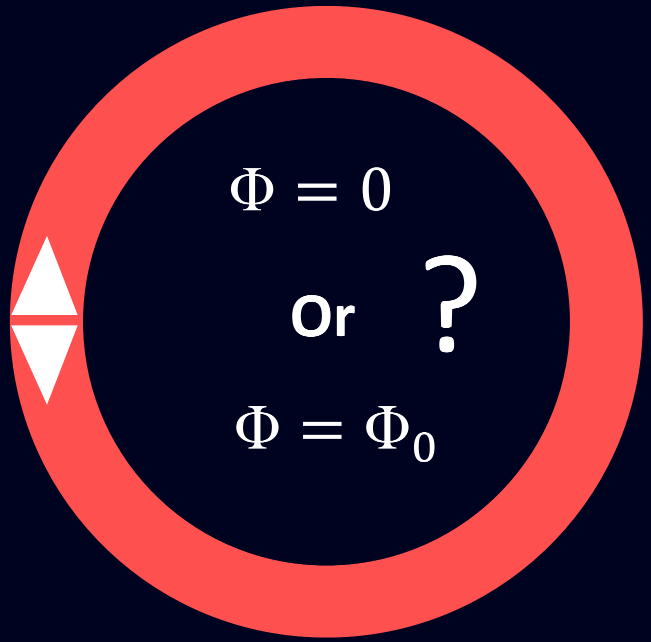
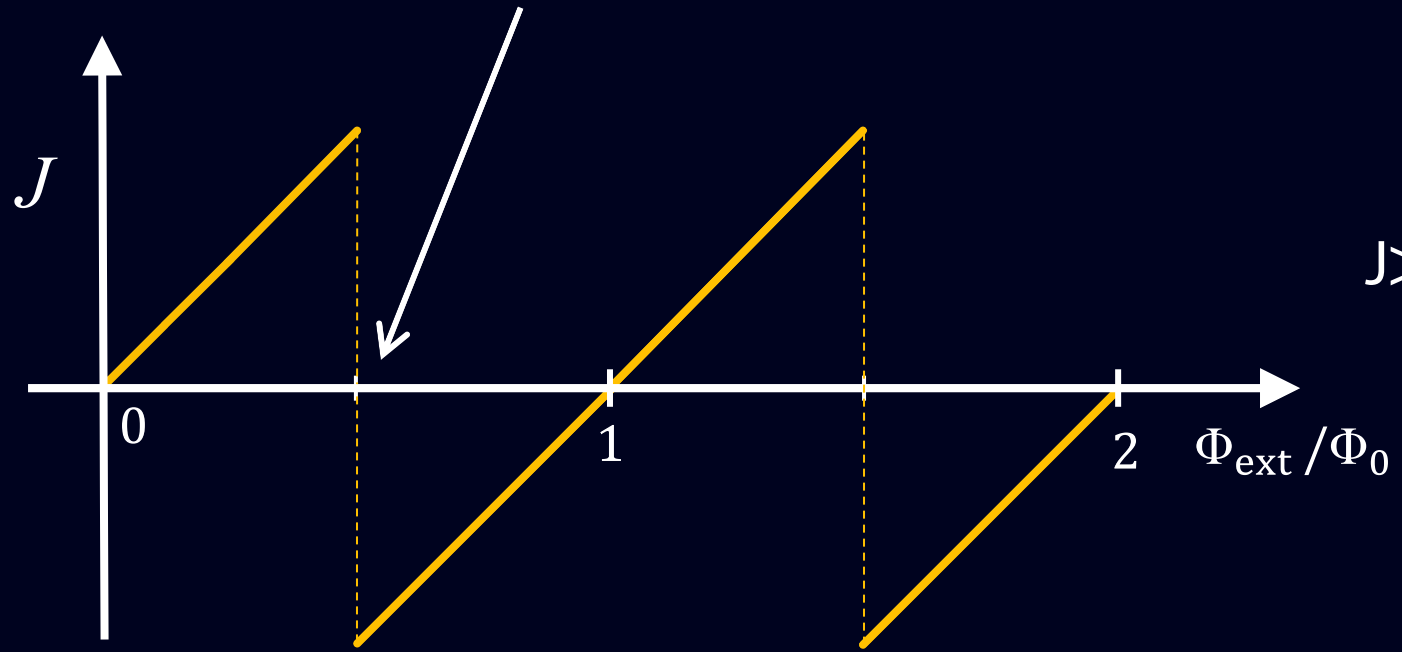


Which way does the supercurrent circulate if Φ_{ext} is exactly $\Phi_0/2$

What's the flux inside the ring? 0 or Φ_0?

J goes both ways at the same time!
The loop is in superposition (i.e., a qubit), where both $\Phi = 0$ and $\Phi = \Phi_0$ happen

What's happening at $\Phi = \Phi_0/2$ exactly?



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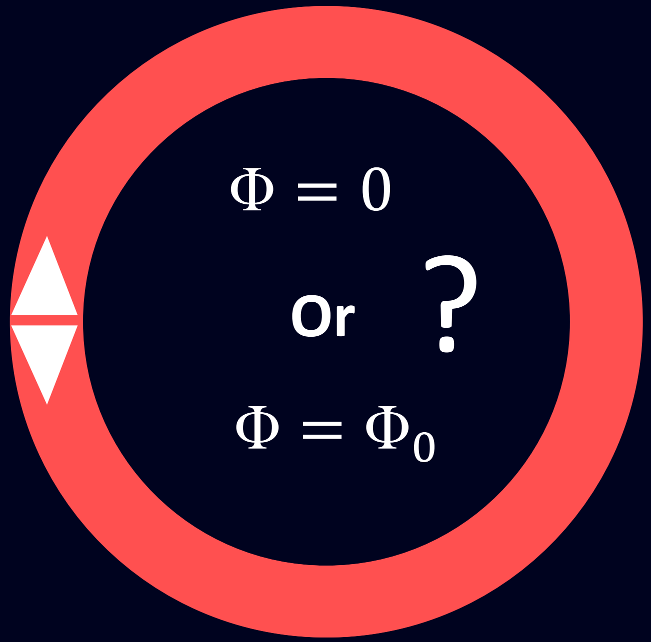
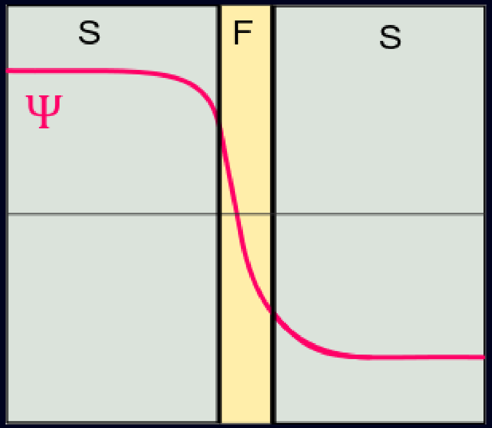
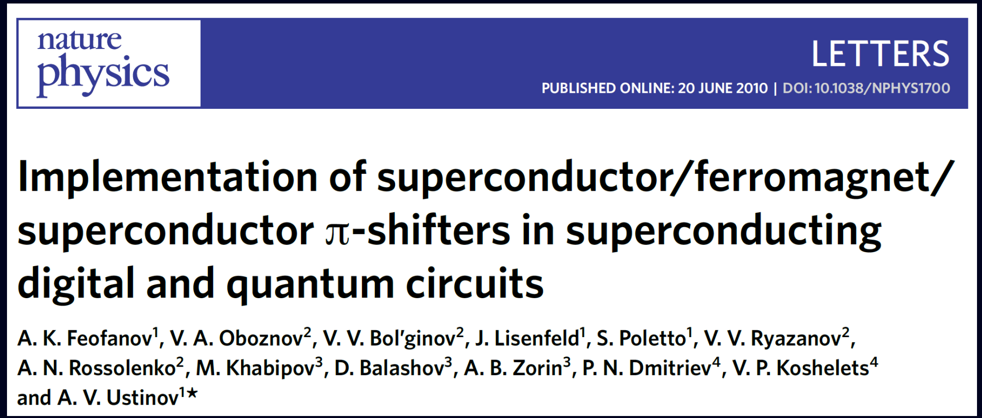


Inserting a π-junction in a loop is equivalent to applying $\Phi_0/2$ flux (see Josephson junctions later)




nature physics
LETTERS
PUBLISHED ONLINE: 20 JUNE 2010 | DOI:10.1038/NPHYS1700

Implementation of superconductor/ferromagnet/superconductor π-shifters in superconducting digital and quantum circuits

A. K. Feofanov[1], V. A. Oboznov[2], V. V. Bol'ginov[2], J. Lisenfeld[1], S. Poletto[1], V. V. Ryazanov[2], A. N. Rossolenko[2], M. Khabipov[3], D. Balashov[3], A. B. Zorin[3], P. N. Dmitriev[4], V. P. Koshelets[4] and A. V. Ustinov[1]*

Break?

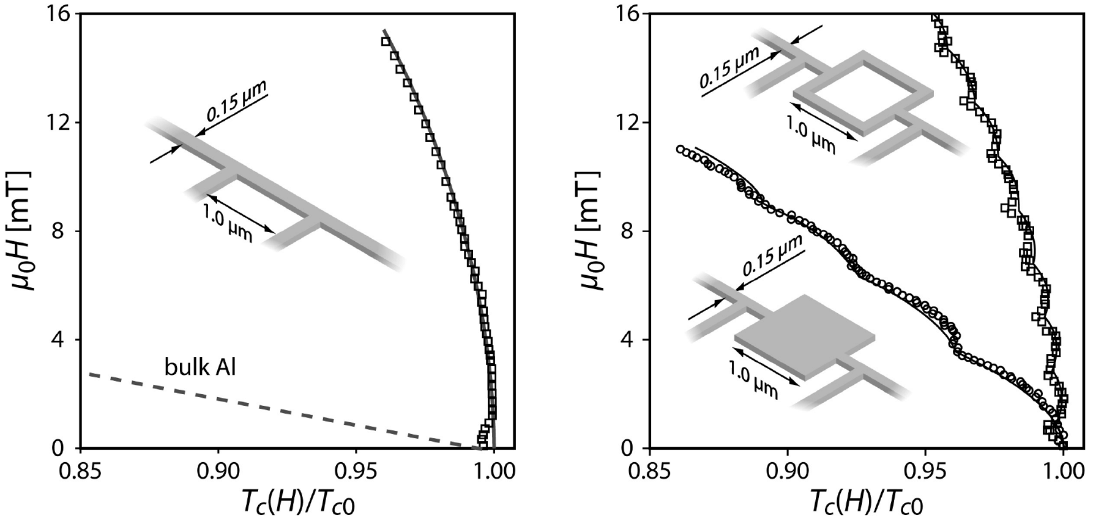
H-T phase diagram: bulk vs mesoscopic

Line

$$T_c(H) = T_{c0}\left[1 - \frac{\pi^2}{3}\left(\frac{w\xi(0)\mu_0 H}{\Phi_0}\right)^2\right]$$

$$T_c(H) \propto H^2$$

The parabolic background corresponds to the London limit, where $|\Psi|$ is constant throughout the mesoscopic structure ($|\Psi|$ is 1D).



(a) line (b) loop and dot

Fig. 2.1 The measured superconducting/normal-state phase boundary as a function of the reduced temperature $T(H)/T_{c0}$ for (a) a line and (b) a loop and a dot. The solid line in (a) is calculated using Eq. (2.1) with $\xi(0) = 110\,\mathrm{nm}$ as a fitting parameter. The dashed line represents $T_c(H)$ for bulk aluminium. Comparing $T_c(H)$ for these three different mesoscopic structures, made of the same material, one clearly sees the effect of topology on $T_c(H)$ (after [300]).

From Moshchalkov

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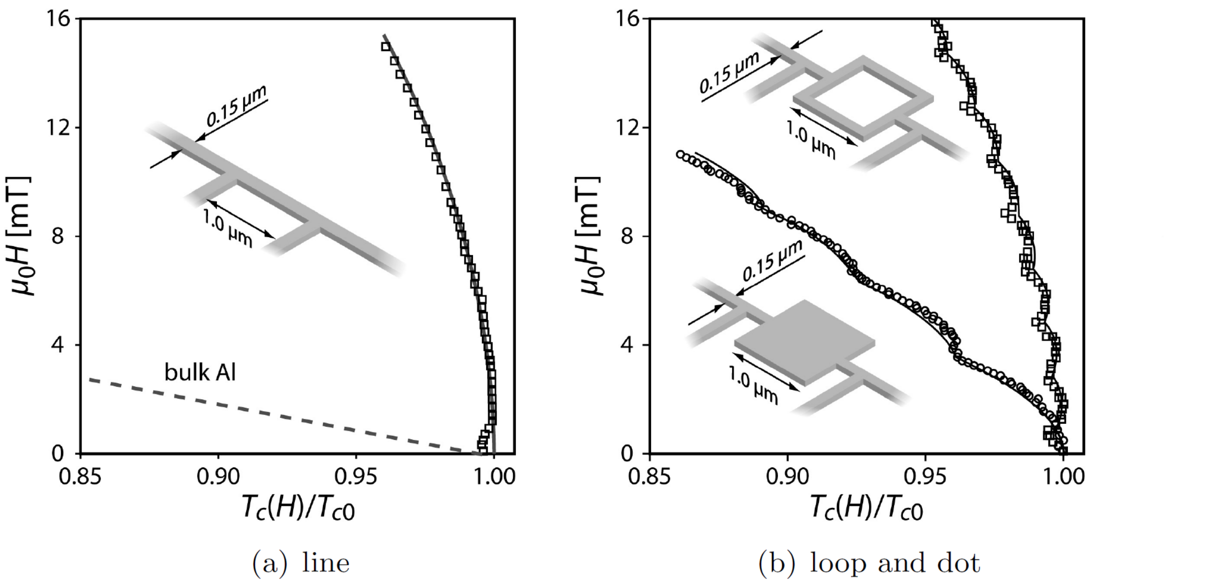
So how "mesoscopic" do you need to be, to be in the London limit?



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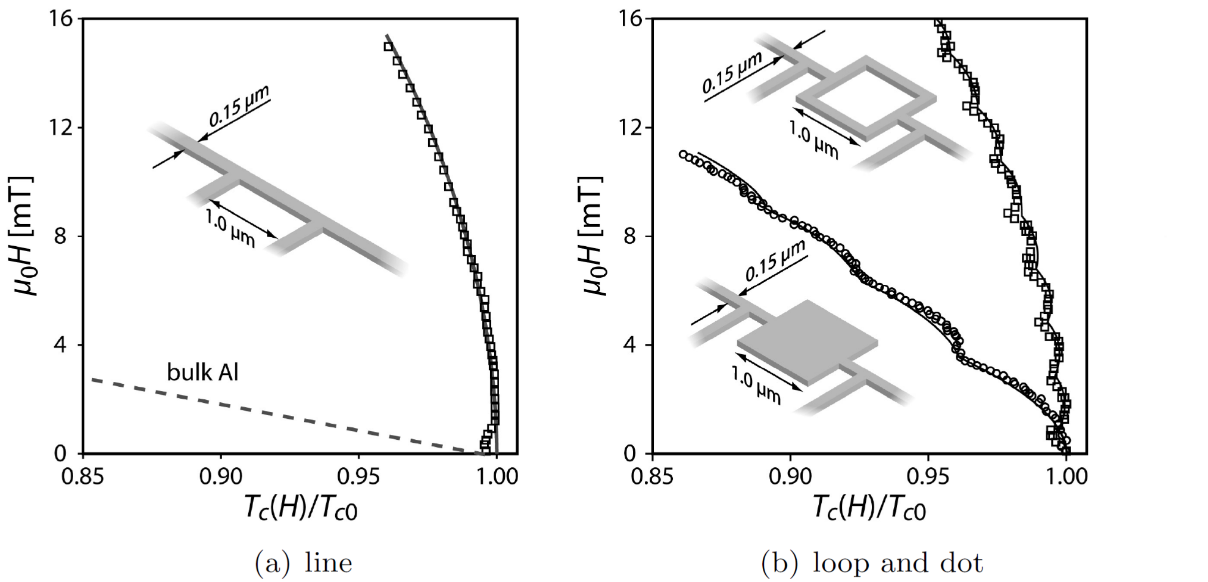

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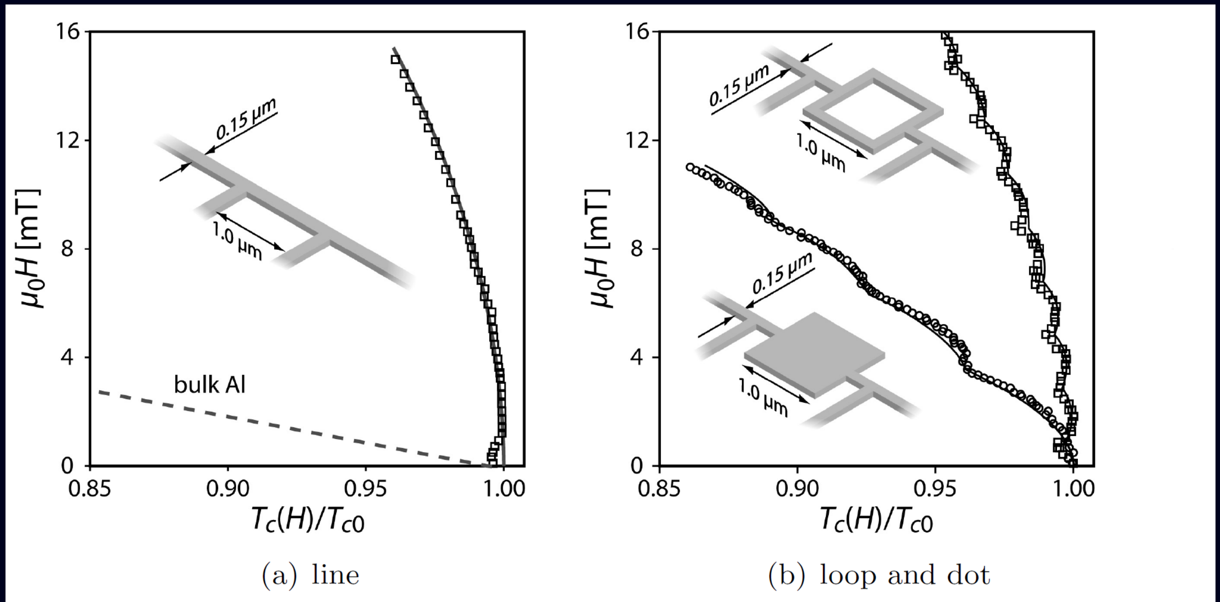
What about the *Hc*-*Tc* of a thin film?



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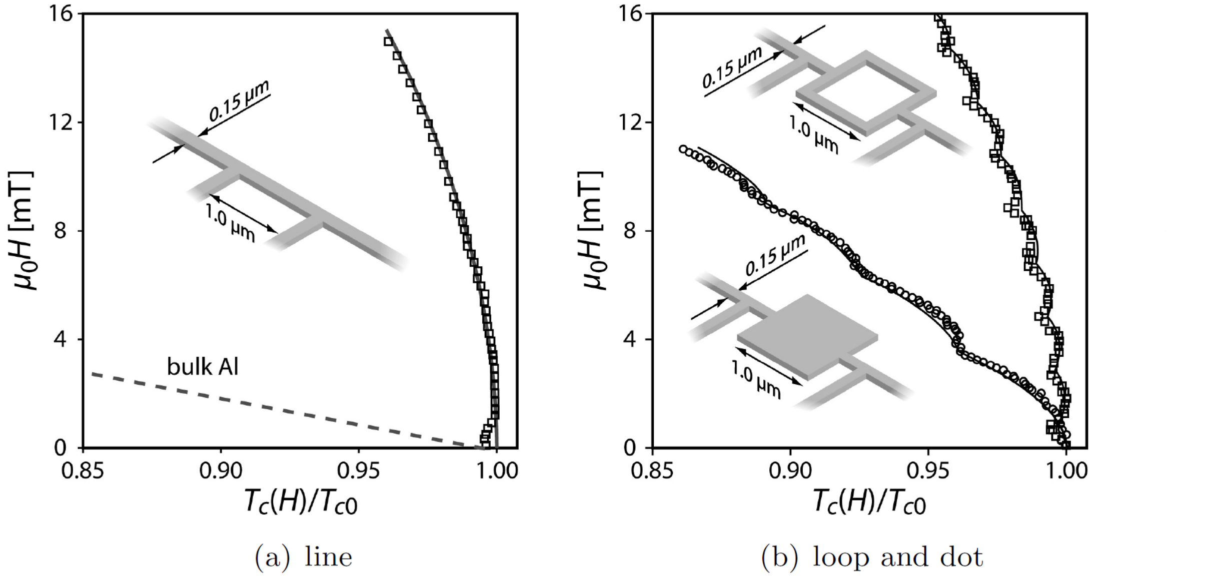
Thin film under in-plane field = Line (1D)



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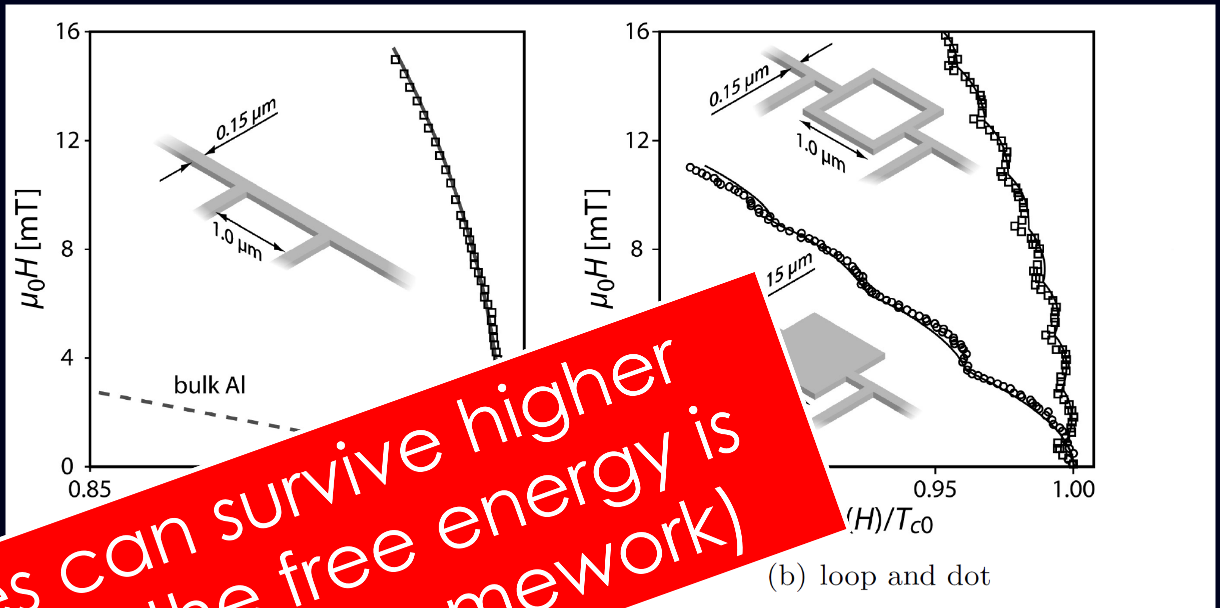

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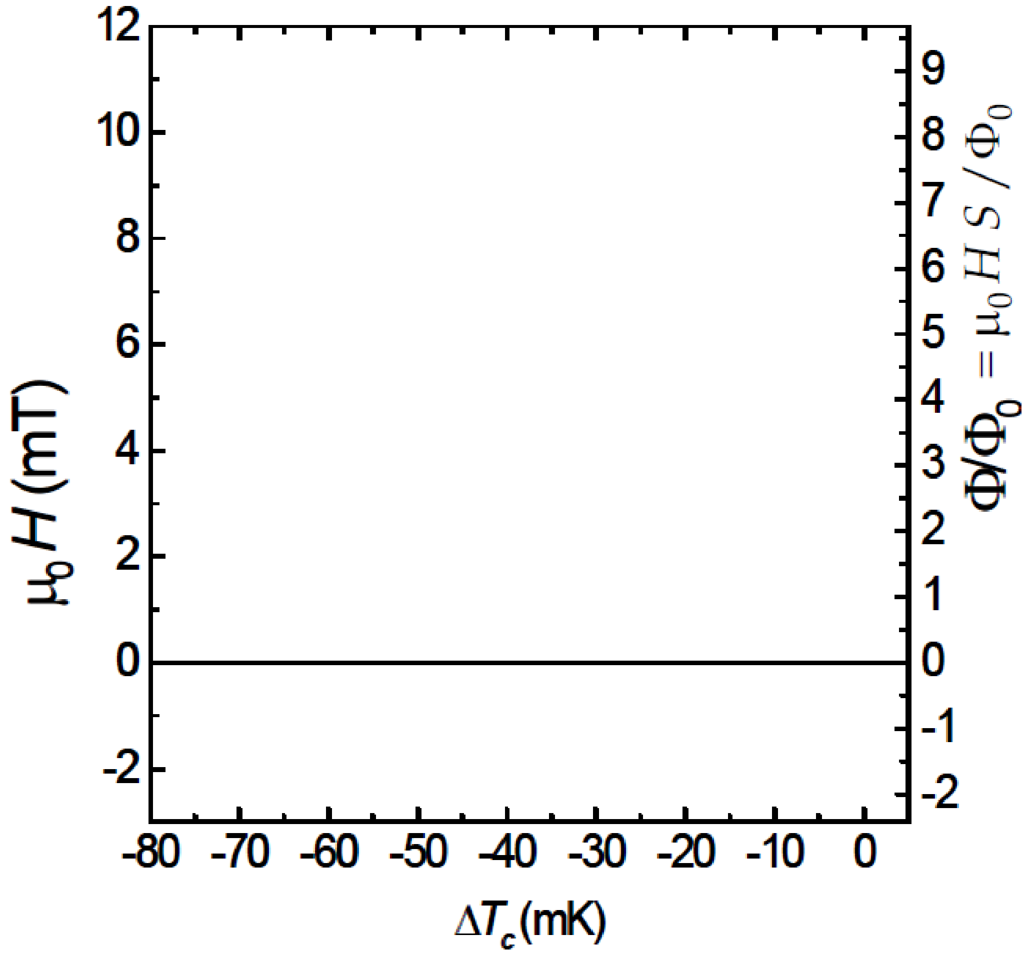
From Moshchalkov

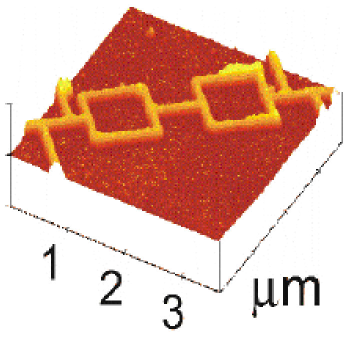
Mesoscopic structures can survive higher magnetic fields because the free energy is lowered by confinement (see homework)

What about the *Hc*-*Tc* of a thin film?

Thin film under in-plane field = Line (1D)

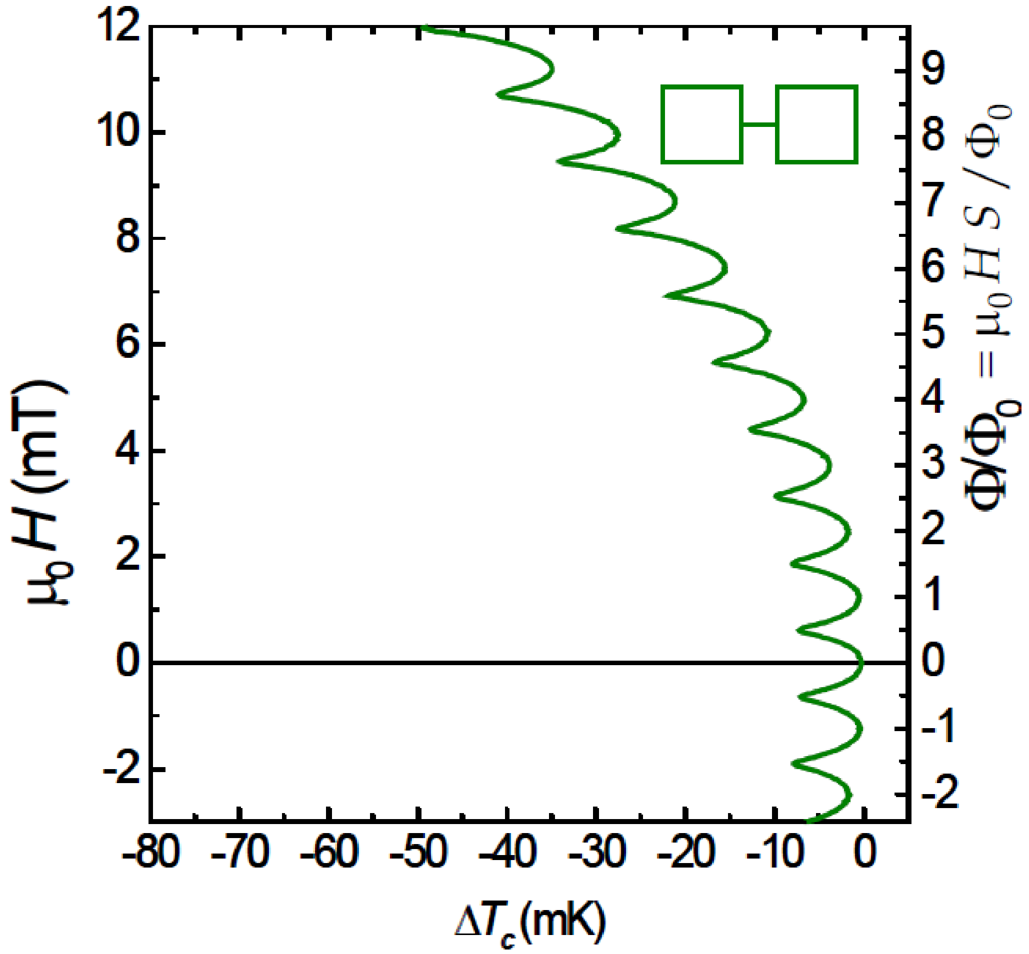
Multi-loop networks

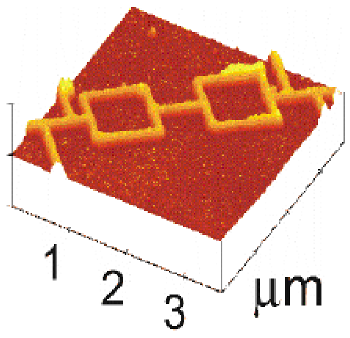




V. Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996).

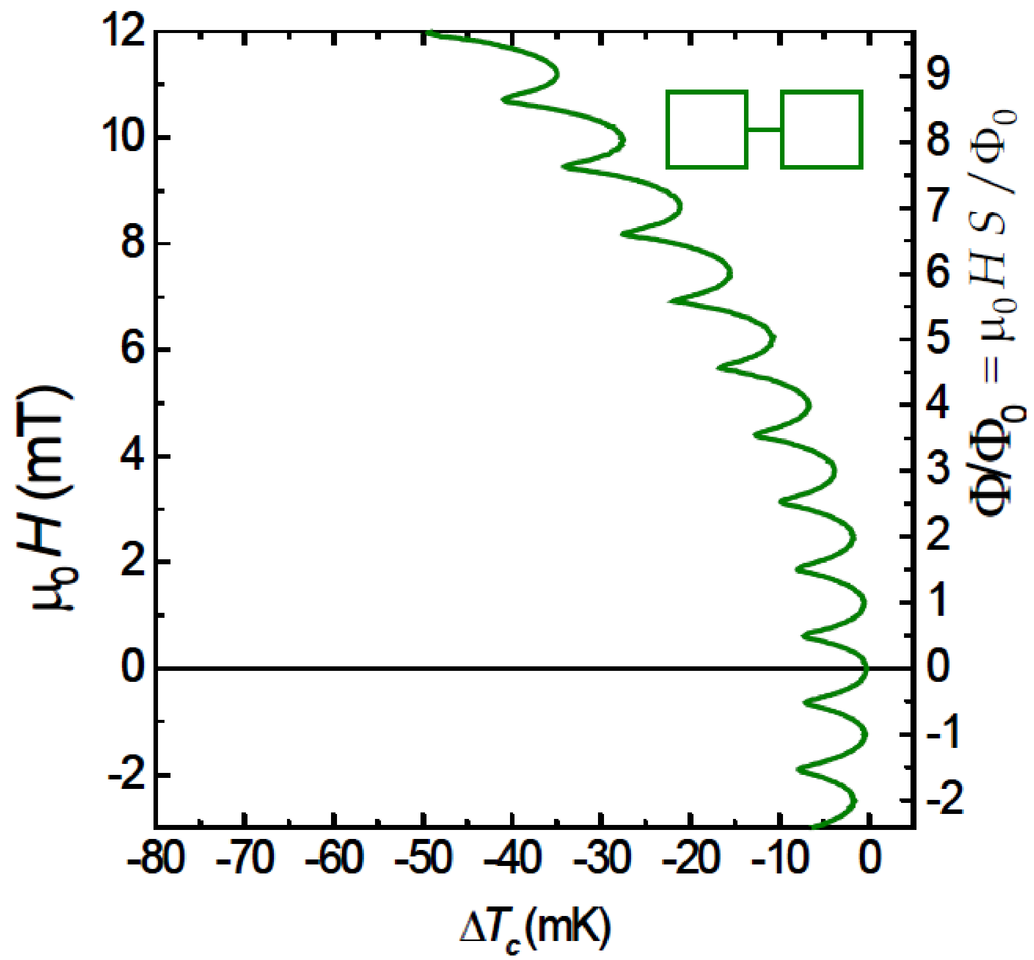
Multi-loop networks

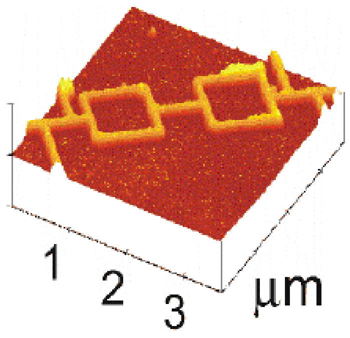


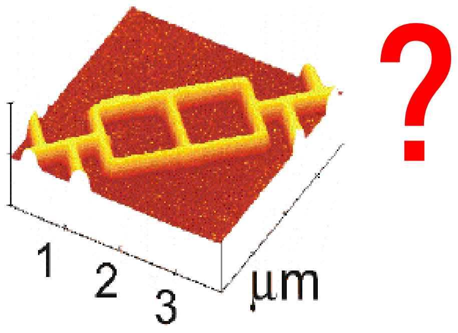


V. Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996).

Multi-loop networks

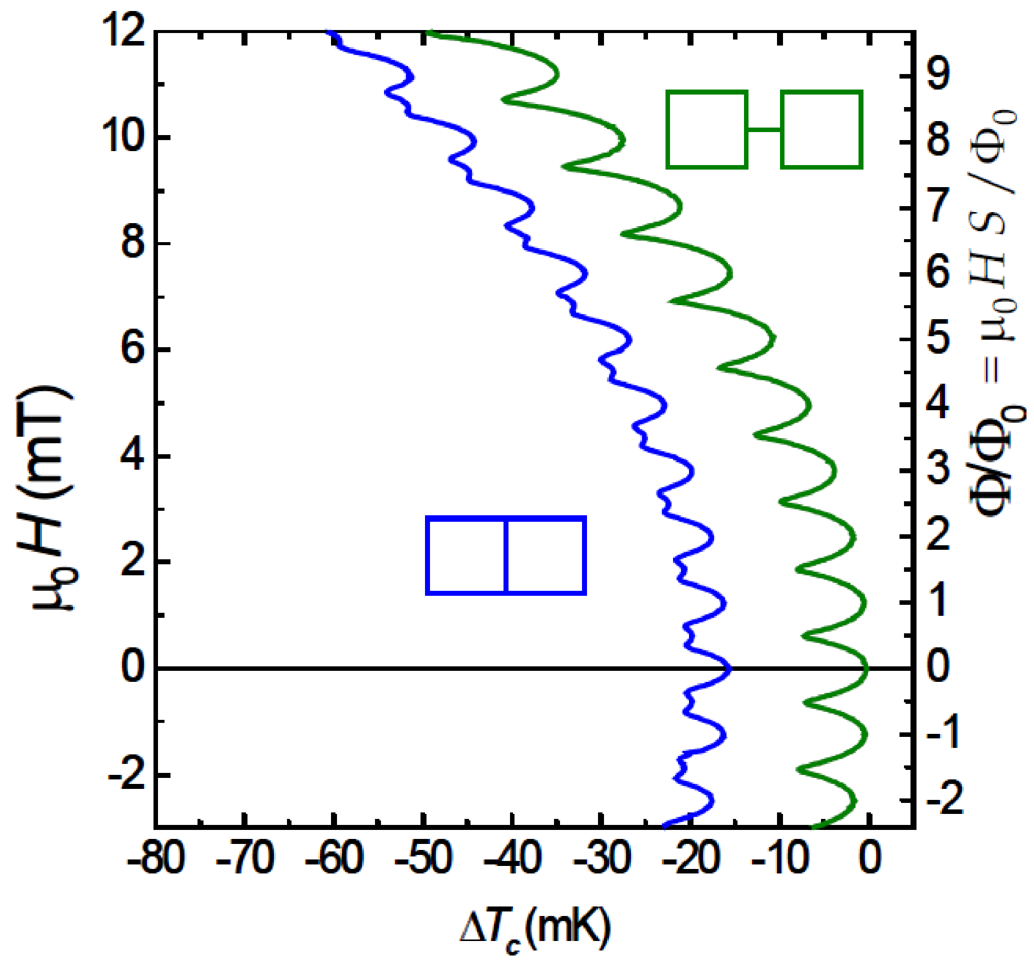


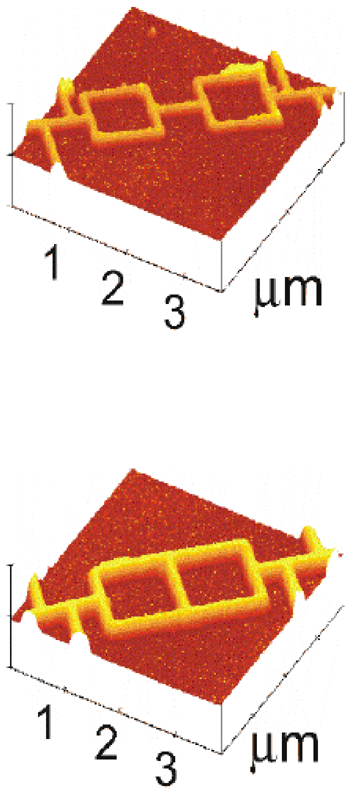




V. Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996).

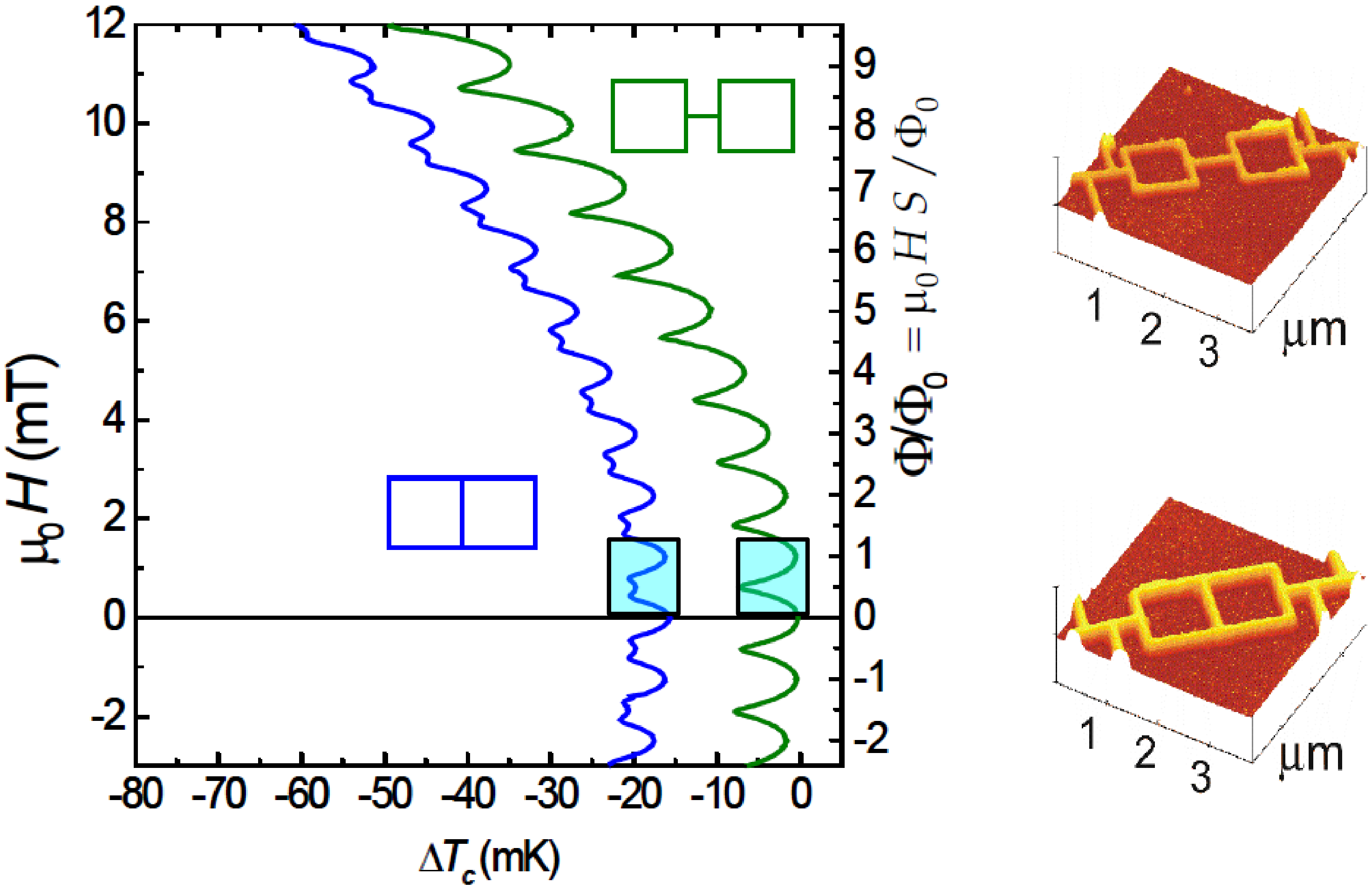
Multi-loop networks





V. Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996).

Multi-loop networks



V. Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996).

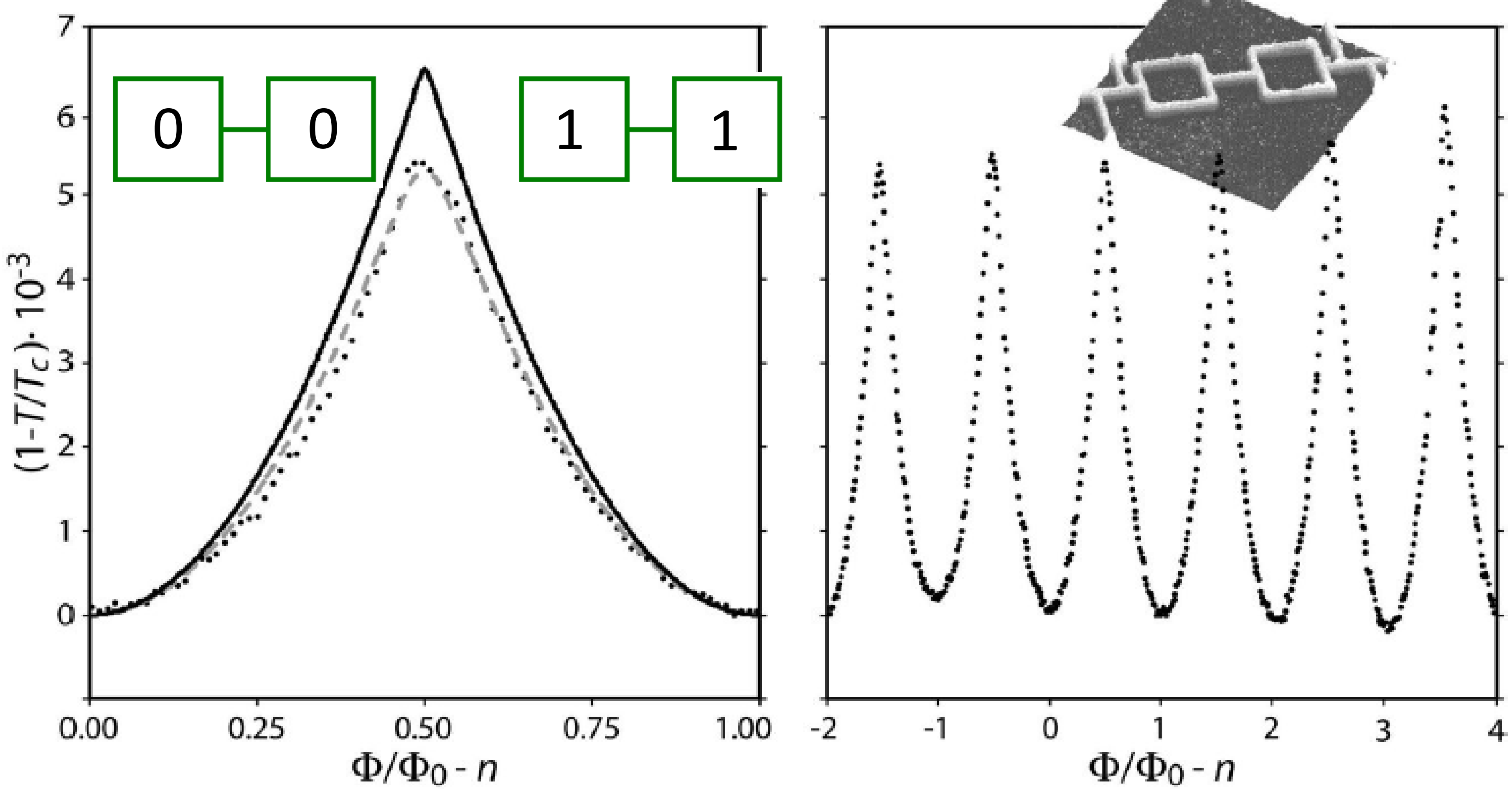
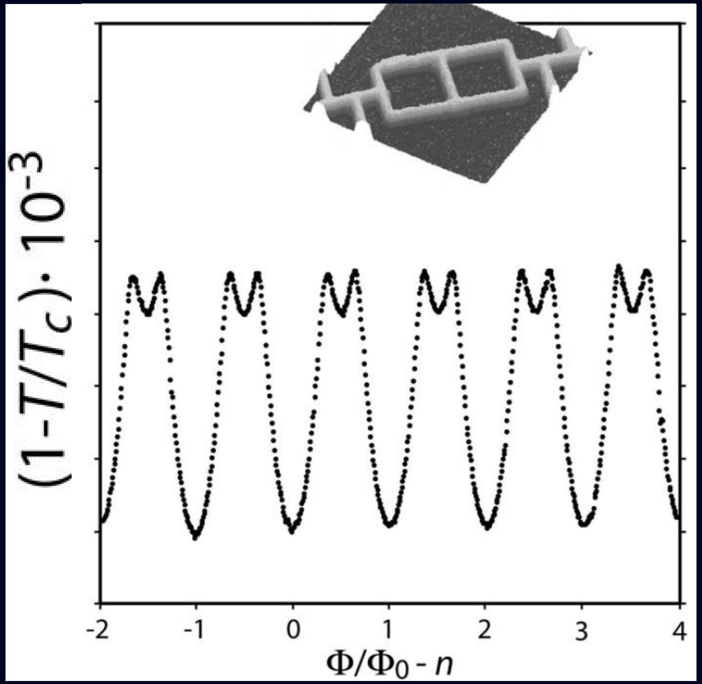
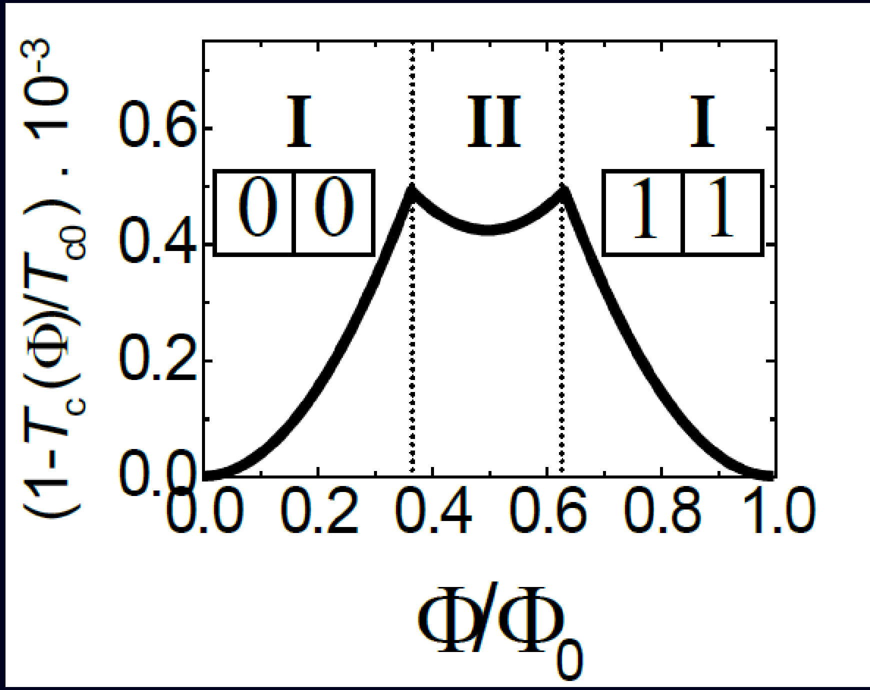


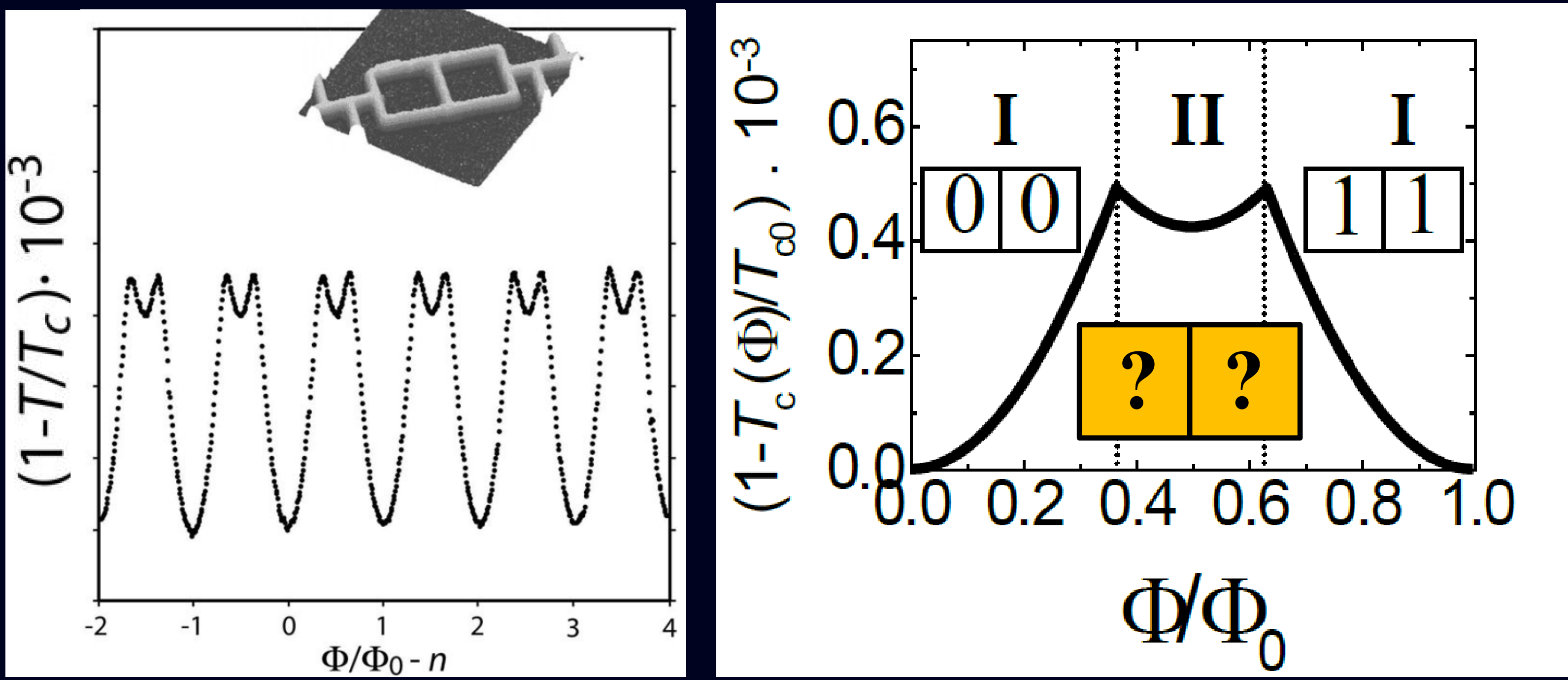
Fig. 3.2 Experimental $T_c(\Phi)$ data for the 'bola' with the parabolic background of Eq. (2.1) subtracted (left and right panels show a single and a few periods, respectively). Experimental data is represented by dots, whereas black and gray lines correspond to the theoretical results obtained in the London limit and with the de Gennes–Alexander approach, respectively. The latter takes the presence of the leads into account (after [345]).

From Moshchalkov

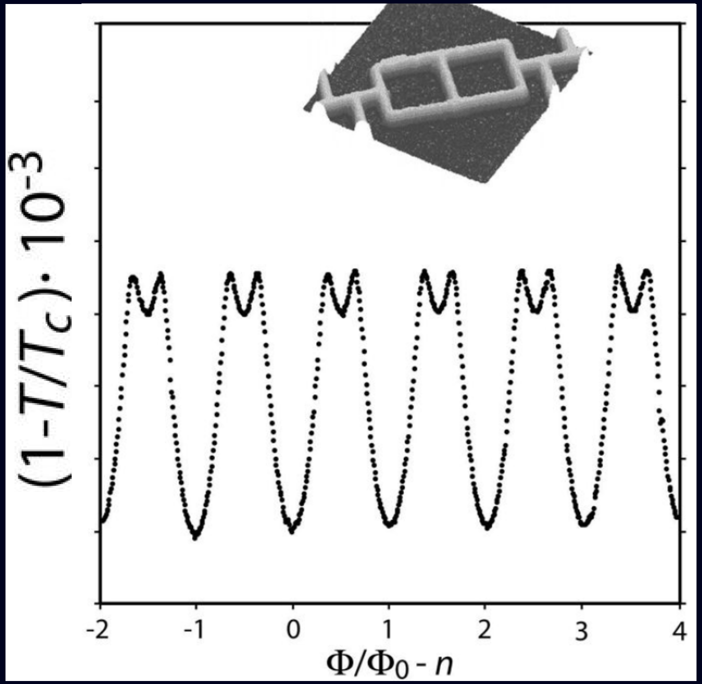


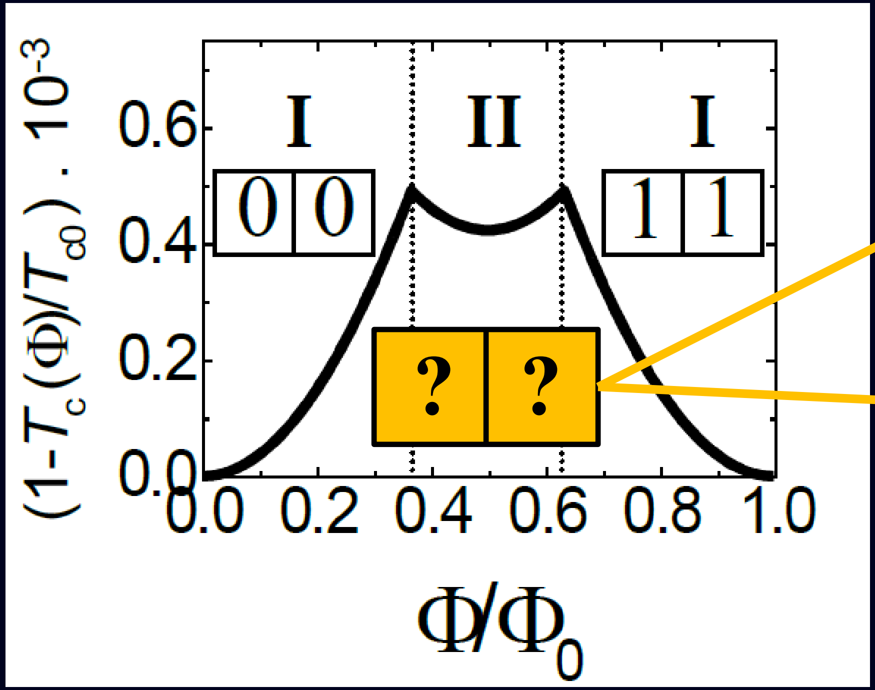


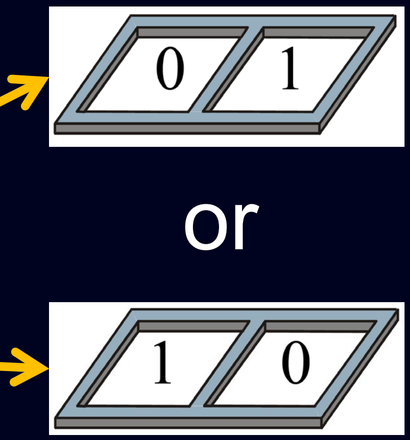
V.Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996)



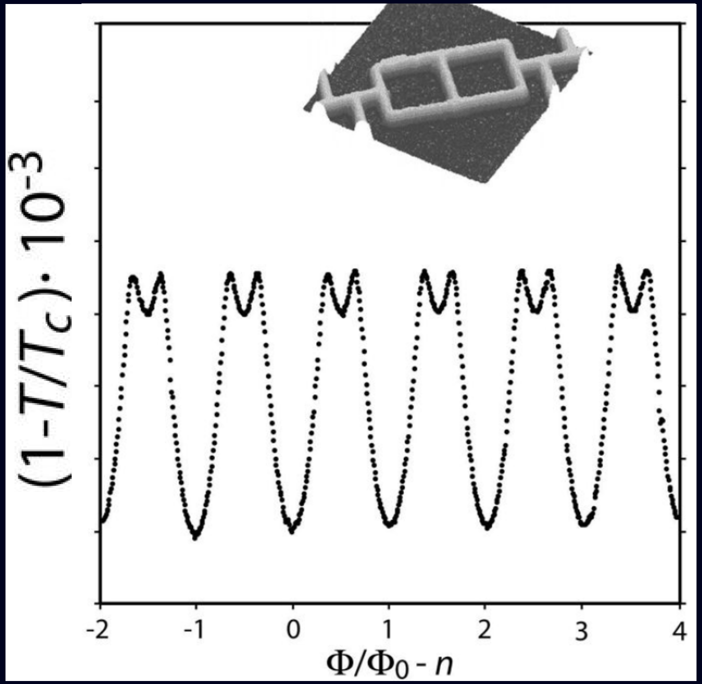
V.Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996)

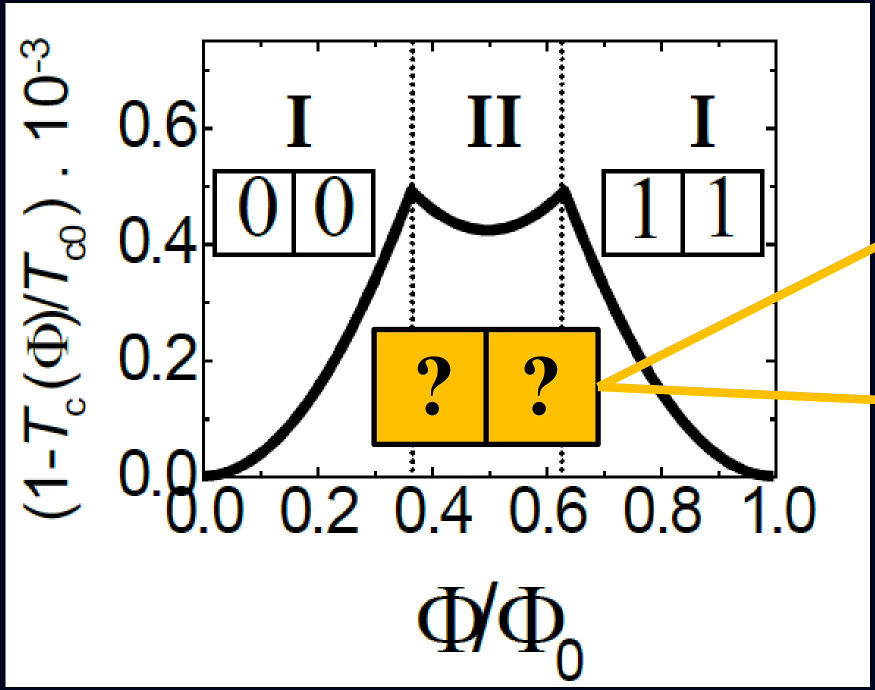


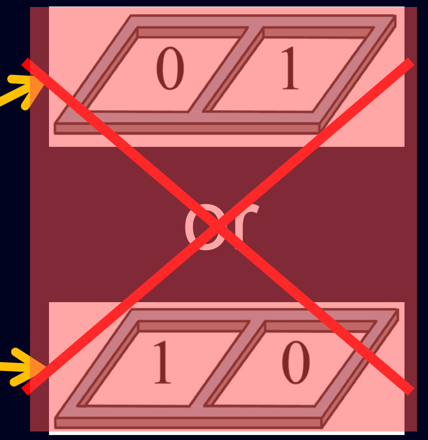




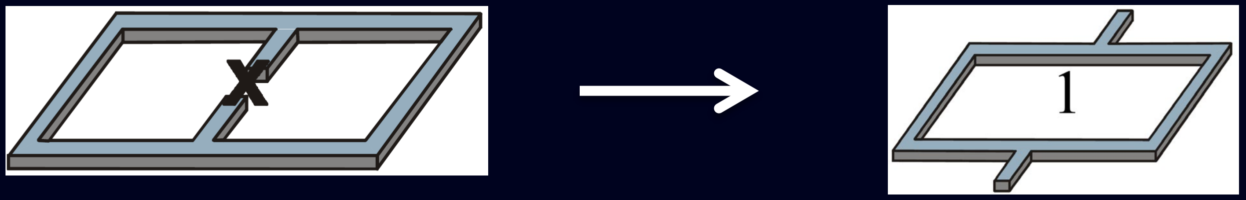
V.Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996)







V.Bruyndoncx *et al.*, Europhys. Lett. **36**,449 (1996)



“a part of the middle link will revert to the normal phase, and that “this in effect will convert the double loop to a single loop”

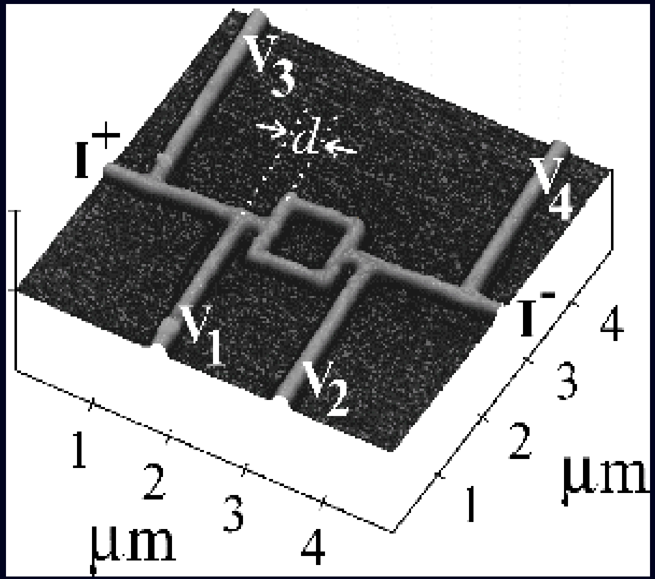
Quantized Magnetic Flux in Superconductors

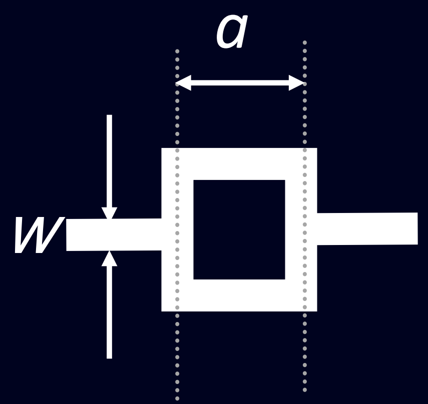
Experiments confirm Fritz London’s early concept that superconductivity is a macroscopic quantum phenomenon.

R. D. Parks

Little-Parks effect in a single mesoscopic Al loop

Strunk et al, PRB, **54**, R12701 (1996)



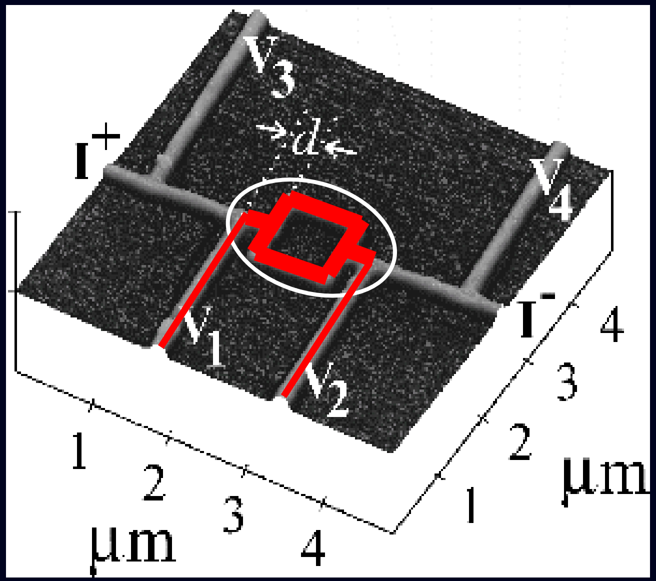


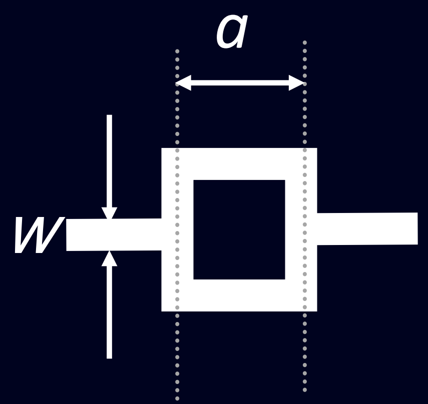
$$\Phi_0 = \frac{h}{2e}$$

$$\Phi = \mu_0 H a^2$$

Little-Parks effect in a single mesoscopic Al loop

Strunk et al, PRB, **54**, R12701 (1996)





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Little-Parks effect in a single mesoscopic Al loop

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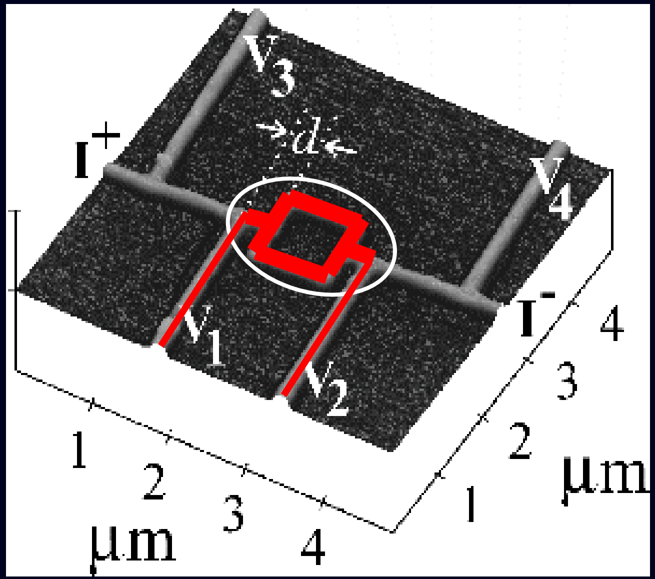
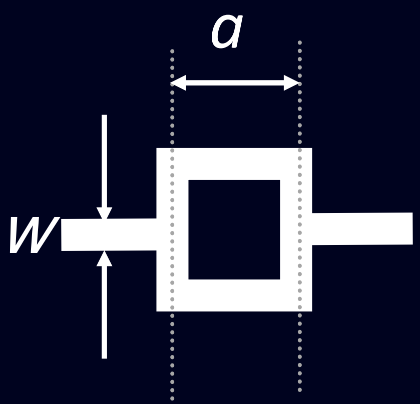
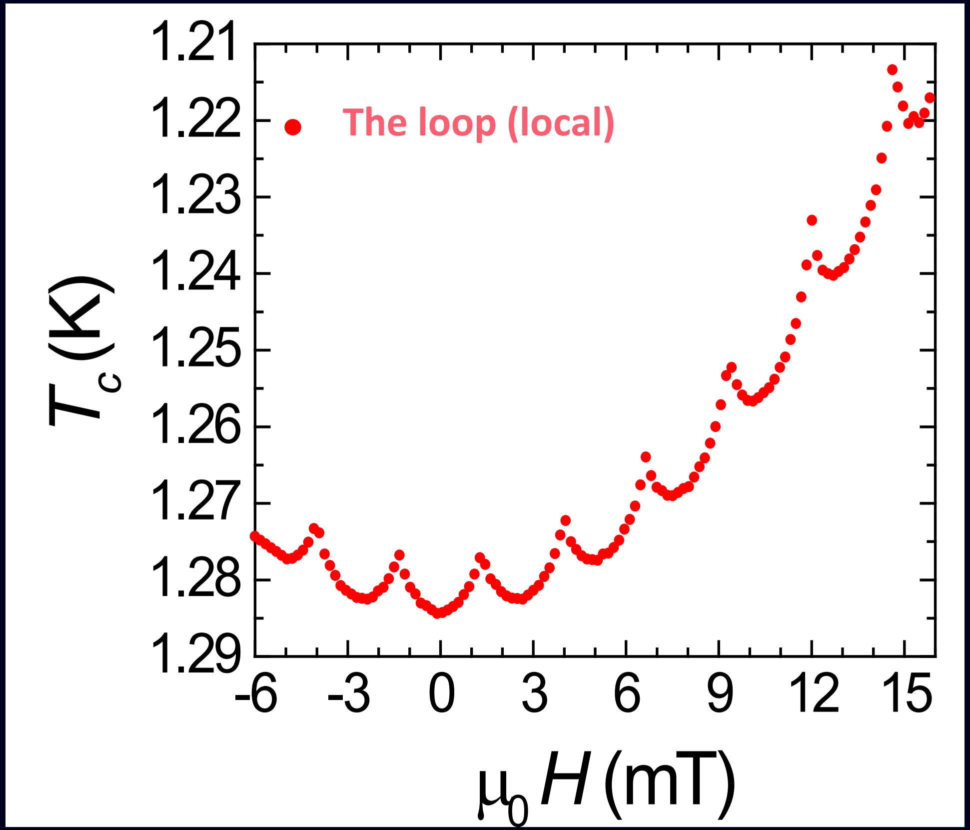




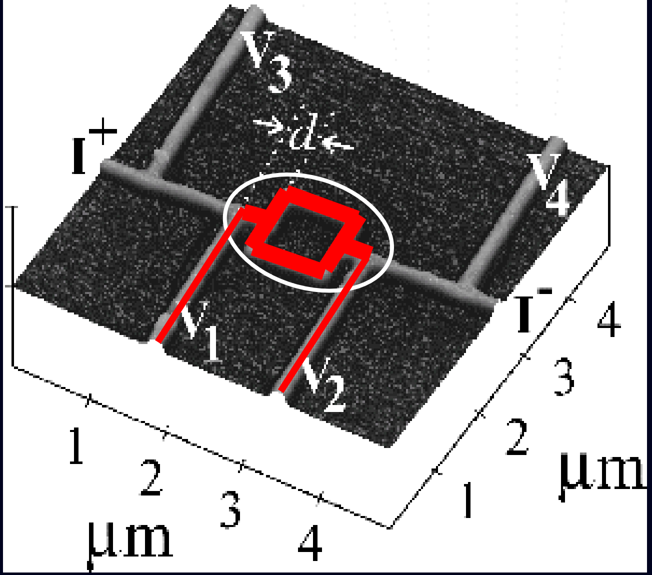
$$\Phi_0 = \frac{h}{2e}$$

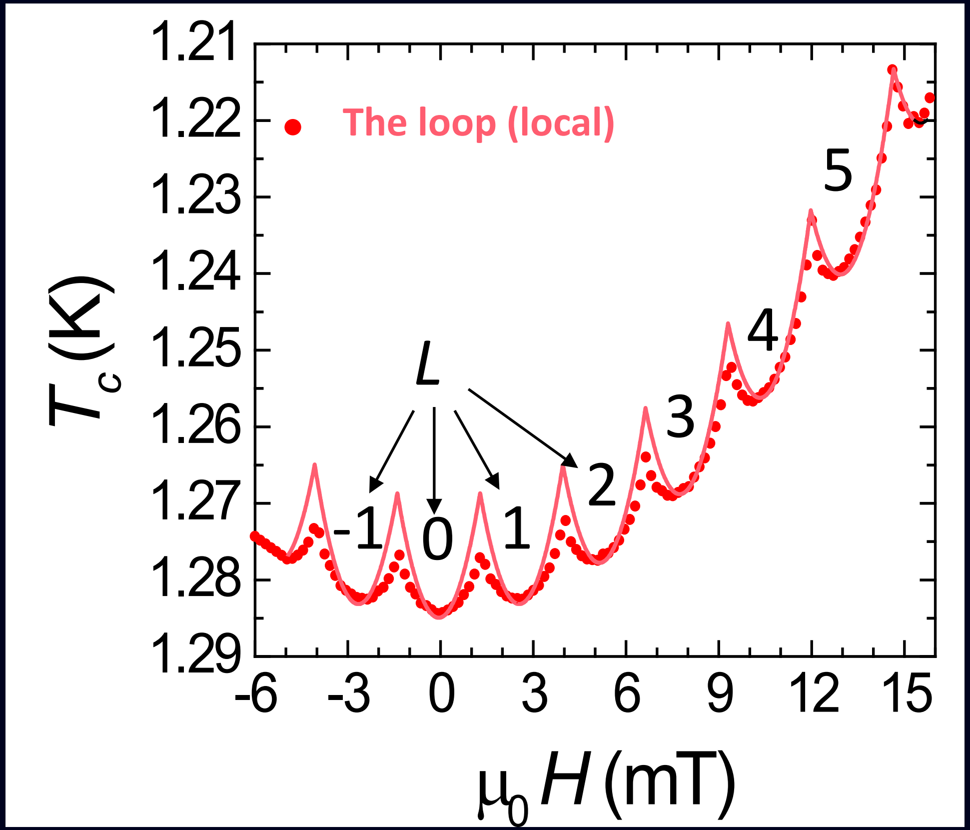
$$\Phi = \mu_0 H a^2$$

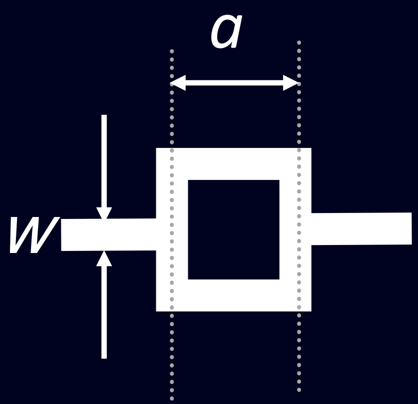


Little-Parks effect in a single mesoscopic Al loop

Strunk et al, PRB, **54**, R12701 (1996)







$$\Phi_0 = \frac{h}{2e}$$

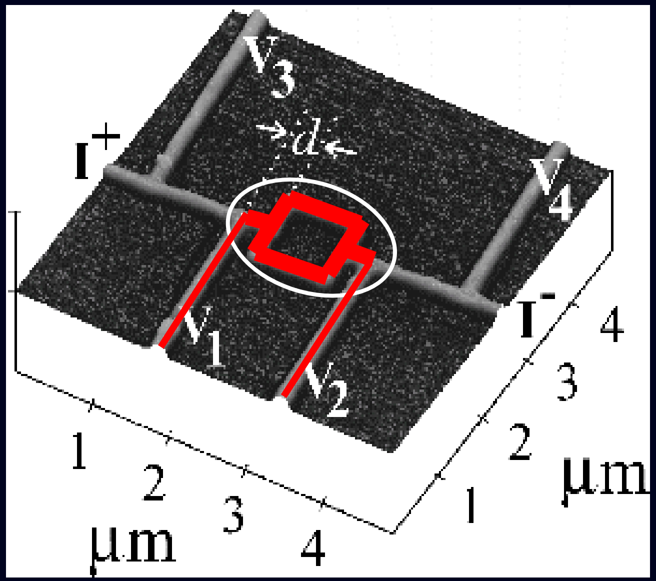
$$\Phi = \mu_0 H a^2$$

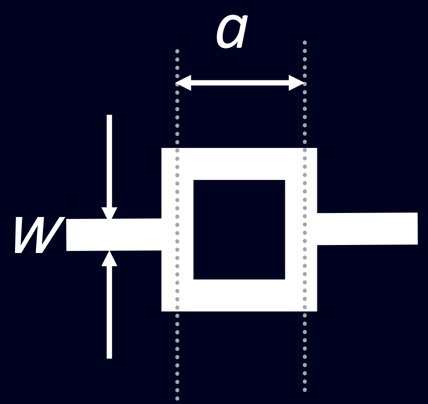
$$\frac{T_{c0} - T_c(\Phi)}{T_{c0}} = \frac{\xi^2(0)\pi^2}{4a^2}\left[\left(L - \frac{\Phi}{\Phi_0}\right)^2 + \frac{4}{3}\frac{w^2}{a^2}\left(\frac{\Phi}{\Phi_0}\right)^2\right]$$

Loop (oscillations)

Little-Parks effect in a single mesoscopic Al loop

Strunk et al, PRB, **54**, R12701 (1996)





$$\Phi_0 = \frac{h}{2e}$$

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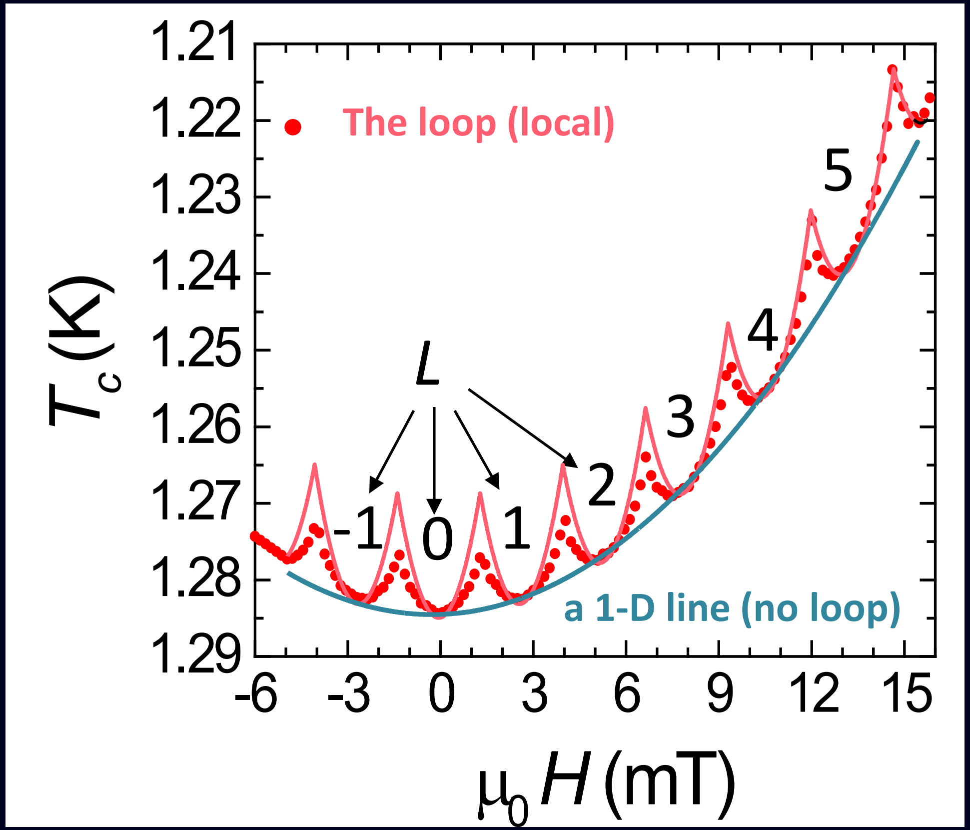


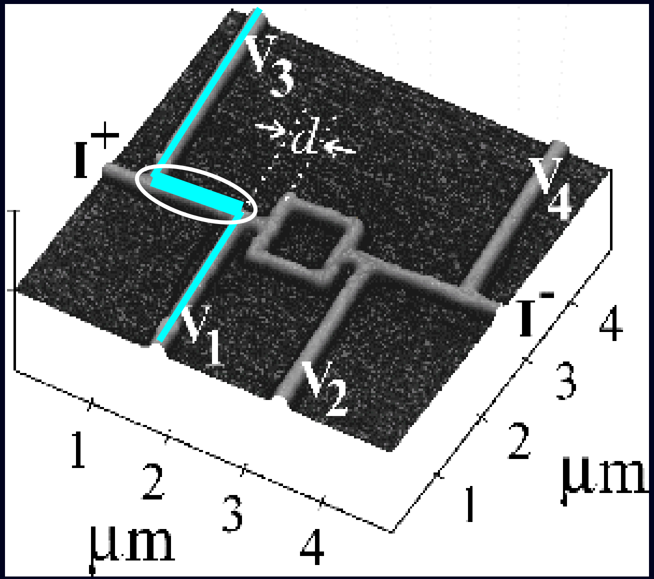
$$\frac{T_{c0} - T_c(\Phi)}{T_{c0}} = \frac{\xi^2(0)\pi^2}{4a^2}\left[\left(L - \frac{\Phi}{\Phi_0}\right)^2 + \frac{4}{3}\frac{w^2}{a^2}\left(\frac{\Phi}{\Phi_0}\right)^2\right]$$

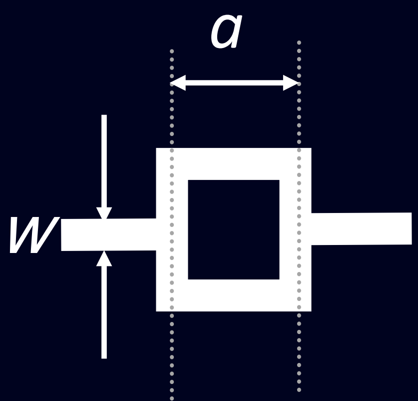
Loop (oscillations)

Parabolic background: Hc (T) in confined SCs

(Non-local) Little-Parks in a mesoscopic Al loop

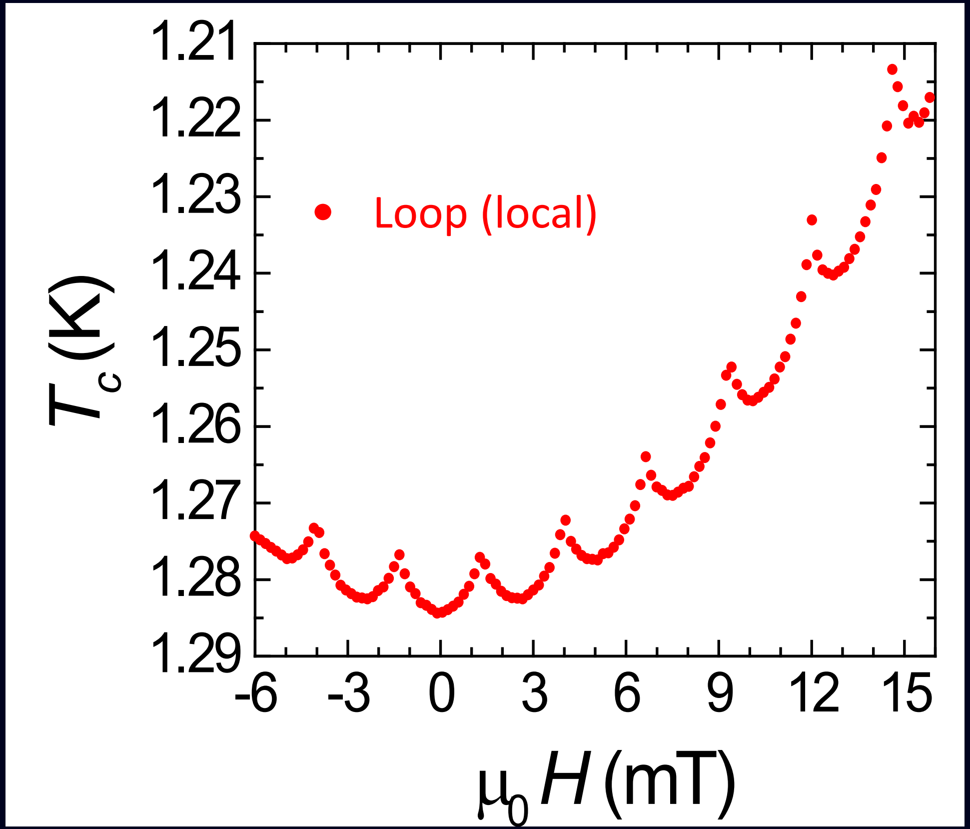
What if we just measured a piece of the connecting wire?





$$\Phi_0 = \frac{h}{2e}$$

$$\Phi = \mu_0 H a^2$$

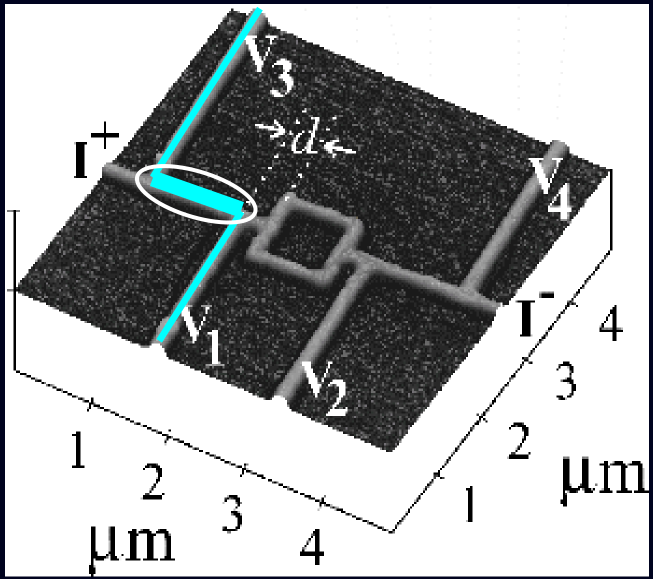


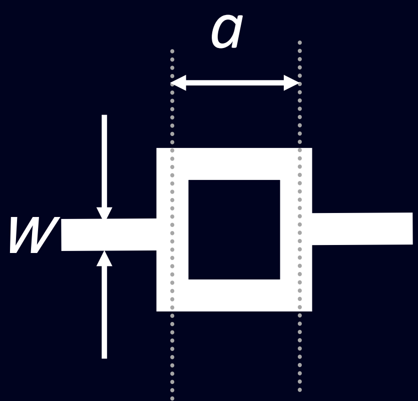
$$\boxed{\frac{T_{c0} - T_c(\Phi)}{T_{c0}} = \frac{\xi^2(0)\pi^2}{4a^2}\left[\left(L - \frac{\Phi}{\Phi_0}\right)^2 + \frac{4}{3}\frac{w^2}{a^2}\left(\frac{\Phi}{\Phi_0}\right)^2\right]}$$

Strunk et al, PRB, **54**, R12701 (1996)

(Non-local) Little-Parks in a mesoscopic Al loop

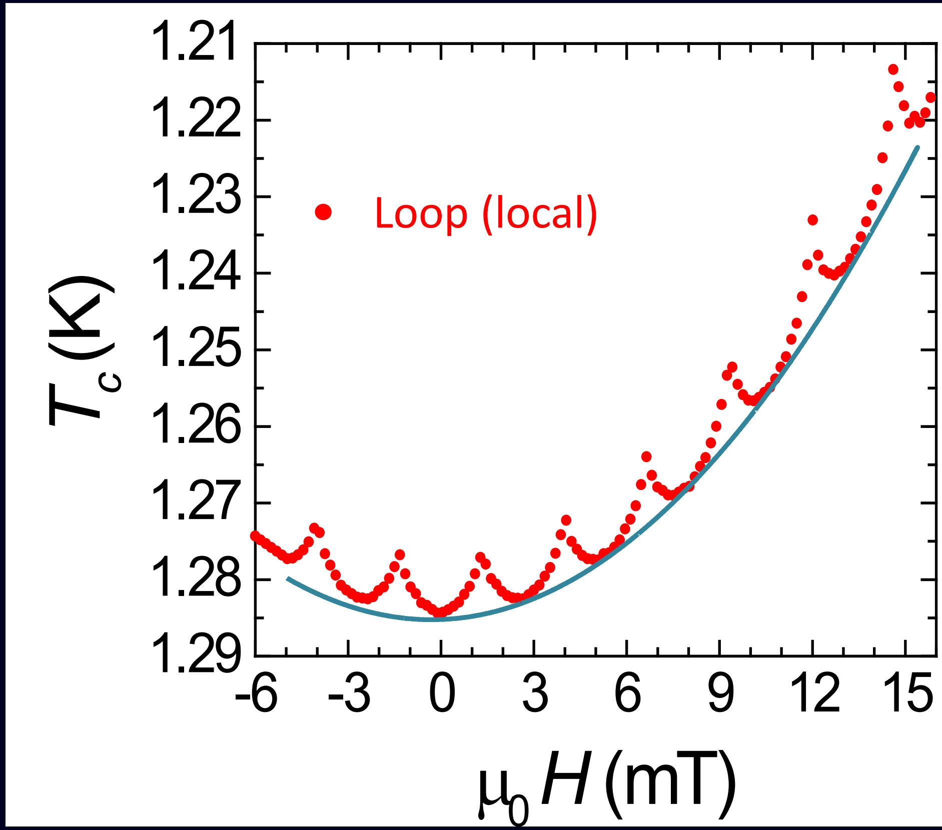
What if we just measured a piece of the connecting wire?





$$\Phi_0 = \frac{h}{2e}$$

$$\Phi = \mu_0 H a^2$$

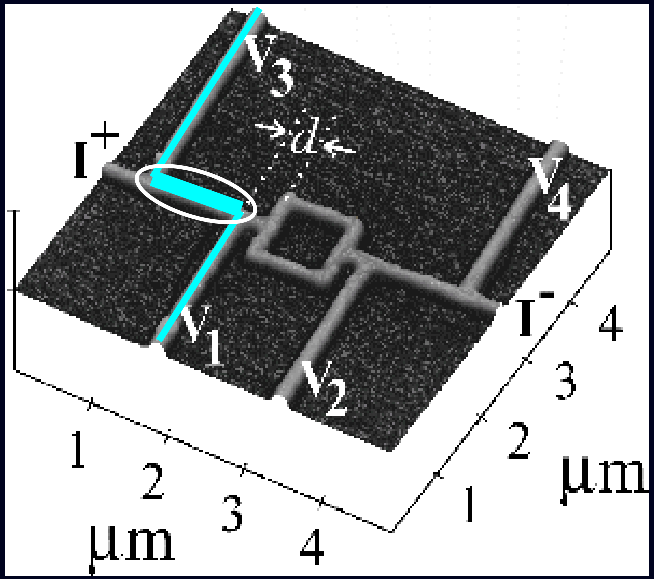


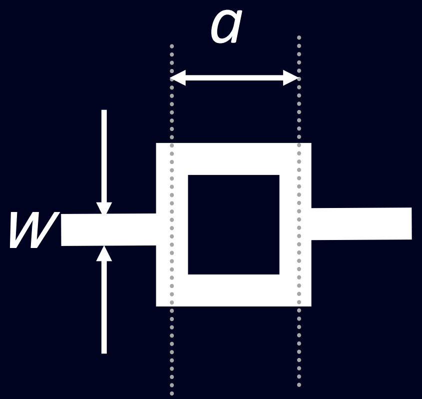
$$\boxed{\frac{T_{c0} - T_c(\Phi)}{T_{c0}} = \frac{\xi^2(0)\pi^2}{4a^2}\left[\cancel{\left(L - \frac{\Phi}{\Phi_0}\right)^2} + \frac{4}{3}\frac{w^2}{a^2}\left(\frac{\Phi}{\Phi_0}\right)^2\right]}$$

Strunk et al, PRB, **54**, R12701 (1996)

(Non-local) Little-Parks in a mesoscopic Al loop

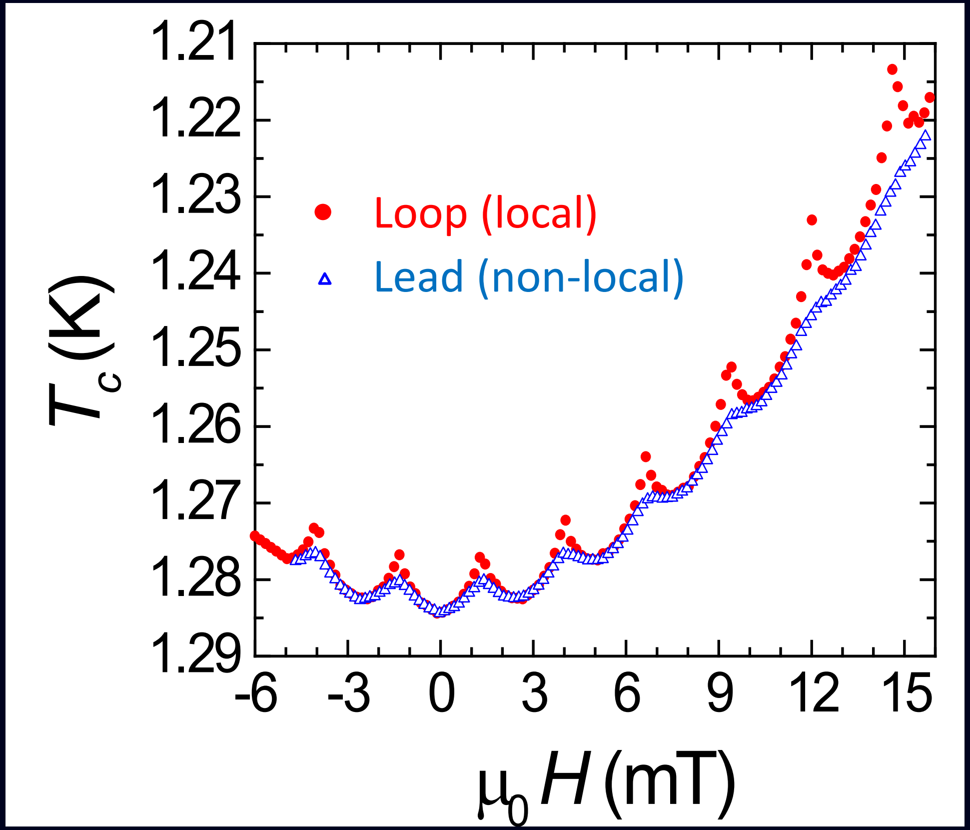
What if we just measured a piece of the connecting wire?





$$\Phi_0 = \frac{h}{2e}$$

$$\Phi = \mu_0 H a^2$$

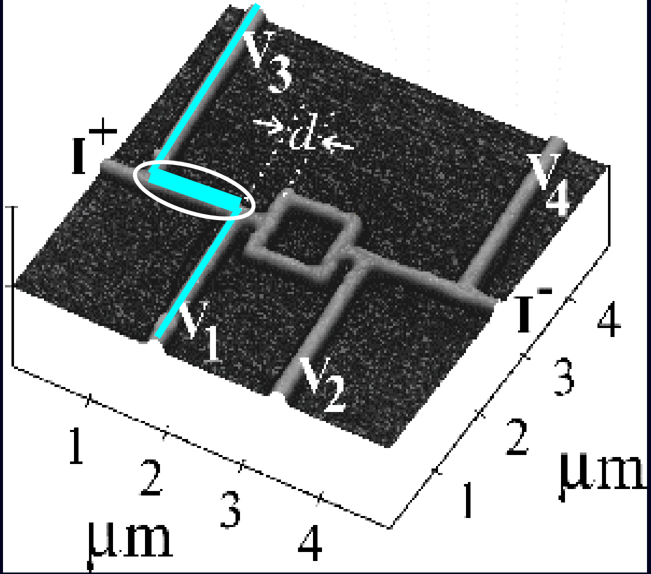


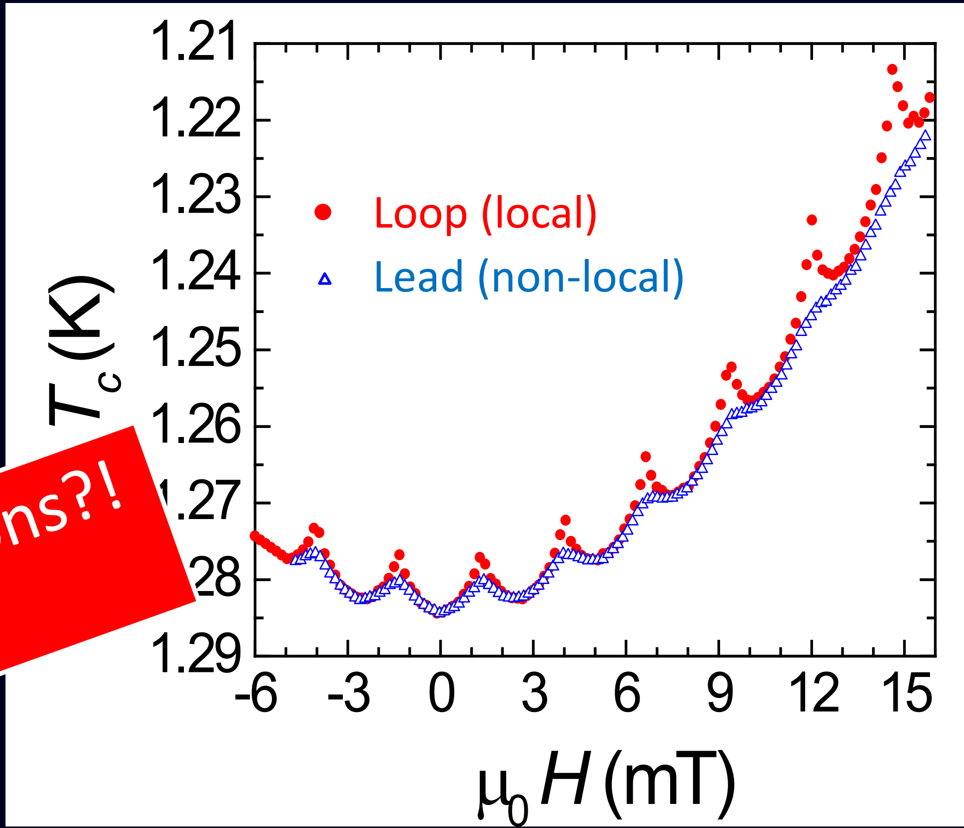
$$\boxed{\frac{T_{c0} - T_c(\Phi)}{T_{c0}} = \frac{\xi^2(0)\pi^2}{4a^2}\left[\left(L - \frac{\Phi}{\Phi_0}\right)^2 + \frac{4}{3}\frac{w^2}{a^2}\left(\frac{\Phi}{\Phi_0}\right)^2\right]}$$

Strunk et al, PRB, **54**, R12701 (1996)

(Non-local) Little-Parks in a mesoscopic Al loop

What if we just measured a piece of the connecting wire?





Why do we still measure the loop oscillations?!
(See homework for more detail)

$$\Phi_0 = \frac{h}{2e}$$

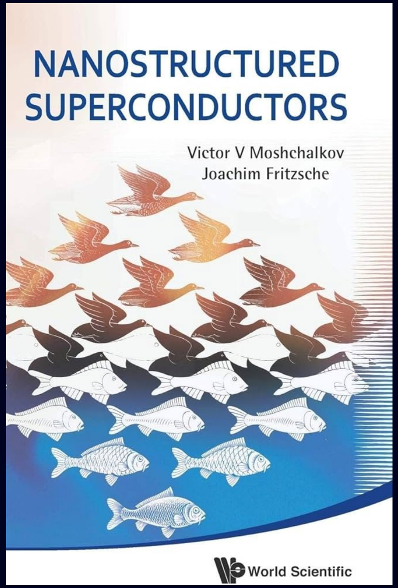
$$\Phi = \mu_0 H a^2$$

$$\boxed{\frac{T_{c0} - T_c(\Phi)}{T_{c0}} = \frac{\xi^2(0)\pi^2}{4a^2}\left[\left(L - \frac{\Phi}{\Phi_0}\right)^2 + \frac{4}{3}\frac{w^2}{a^2}\left(\frac{\Phi}{\Phi_0}\right)^2\right]}$$

Strunk et al, PRB, **54**, R12701 (1996)

End of Lecture 2

A lot of the material covered in this lecture can be found on Moshchalkov's book



Kaveh Lahabi (2025)